B.TECH/ AEIE/BT/CE/CHE/ CSE /ECE/ EE/ IT/ ME /1ST SEM/ MATH 1101 /2018

MATHEMATICS-I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If A be an orthogonal matrix, then
(a) $|A|=2$ (b) $|A|=\pm 1$ (c) $|A|=0$ (d) $|A|=\pm 3$.

(ii) If $f=2x^2-3y^2+4z^2$, then *curl(grad f)*=
(a) $4x\hat{i}-6y\hat{j}+8z\hat{k}$ (b) 3 (c) $x\hat{i}+y\hat{j}+z\hat{k}$ (d) 0.

(iii) Which one of the following is a divergent series?
(a) $\sum_{n=1}^{\infty}\frac{1}{n^4}$ (b) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (c) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}$ (d) $\sum_{n=1}^{\infty}\frac{1}{2^n}$.

(iv) The series $\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\ldots\ldots$ is
(a) convergent (b) divergent
(c) oscillatory (d) conditionally convergent.

(v) The complementary function of the differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$ is
(a) $Ae^{-2x}+Be^{-x}$ (b) $Ae^{2x}+Be^{x}$ (c) $Ae^{2x}+Be^{x}$ (d) $Ae^{-2x}+Be^{x}$

(vi) Degree and order of the differential equation $\left(\frac{d^2x}{dt^2}\right)^2+\frac{d^2x}{dt^2}+t\frac{dx}{dt}=0$ are
(a) 2,1 (b) 2,2 (c) 1,1 (d) 1,2

(vii) If f (x , y)=0 then $\frac{dy}{dx}$ is equal to
(a) $-\frac{fx}{fy}$ (b) $\frac{fx}{fy}$ (c) $\frac{fy}{fx}$ (d) $-\frac{fx}{fy}$.

(viii) If $\vec{u}\times\frac{d\vec{u}}{dt}=\vec{0}$, then $\vec{u}(t)$ is of
(a) constant magnitude (b) constant direction
(c) constant magnitude and direction (d) none of these.

(ix) if $u=f\left(y/x\right)$, then $xu_x + yu_y$ is
(a) 0 (b) 2 (c) 3 (d) x + y.

(x) The value of $\int_1^0\int_0^1 (x+y)dxdy$ is
(a) 2 (b) 3 (c) -1 (d)0.

Group – B

2. (a) Determine the rank of the following matrix:
$$\begin{bmatrix}2 & -1 & 3 & 4\\ 0 & 3 & 4 & 1\\ 2 & 3 & 7 & 5\\ 2 & 5 & 11 & 6\end{bmatrix}$$

(b) Is the matrix $A=\begin{bmatrix}1 & 2 & 1\\ -1 & 0 & 2\\ 2 & 1 & 3\end{bmatrix}$ equivalent to I_3? Justify.

(c) If λ is a non-zero eigen value of an invertible matrix A, then show that $1/\lambda$ is an eigen value of A^{-1}.

4 + 5 + 3 = 12

3. (a) Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}1 & -2 & 2\\ 1 & 2 & 3\\ 0 & -1 & 2\end{bmatrix}$. Use the theorem to find A^{-1}.

(b) Determine the values of *a* and *b* for which the system of equations
$$x+2y+3z=6$$
$$x+3y+5z=9$$
$$2x+5y+az=b$$
has (i) no solution, (ii) unique solution, (iii) infinite number of solutions.

6 + 6 = 12

Group – C

4. (a) Test the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt[3]{3n^2+1}}{\sqrt[4]{4n^3+2n+7}}$

(b) Find the directional derivative of the scalar function f (x, y, z)= $x^2 + xy + z^2$ at the point A (1, –1, –1) in the direction of the line AB where B has co-ordinates (3, 2, 1).

6 + 6 = 12

5. (a) Test the convergence of $\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + ...$

(b) Find the constant a, so that $\vec{A} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational. Further, show that $\vec{A}$ can be shown as the gradient of a scalar function.

6 + 6 = 12

Group – D

6. (a) Solve $(2x \log x - xy)dy + 2ydx = 0$.

(b) Solve the given differential equation by the method of variation of parameters

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2.$$

5 + 7 = 12

7. (a) Solve $y = 2px + p^n, p = \frac{dy}{dx}$.

(b) Solve the following differential equation by using D- operator method

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

6 + 6 = 12

Group – E

8. (a) Verify Green's theorem in a plane for $\oint_C (y - \sin x)dx + \cos x\, dy$, where C represents the triangle with vertices (0, 0), $(\pi/2, 0), (\pi/2, 2)$.

(b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = (1 - 4\sin^2 u)\sin 2u.$$

6 + 6 = 12

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9. (a) Evaluate the integral $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$, by a change in the order of integration.

(b) Evaluate the double integral $\iint \frac{x^2y^2}{x^2+y^2} dxdy$ over the annular region between the circles $x^2 + y^2 = 4,\ x^2 + y^2 = 9$.

6 + 6 = 12