OPERATIONS RESEARCH
(MECH 4103)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The distinguishing feature of an LP model is
(a) relationship among all variables is linear
(b) it has single objective function and constraints
(c) value of decision variables is non-negative
(d) all of the above.

(ii) If two constraints do not intersect in the positive quadrant of the graph in LP, then
(a) the problem is infeasible (b) the solution is unbounded
(c) one of the constraints is redundant (d) none of the above.

(iii) The probabilistic time is given by (where t_o = Optimistic time, t_p = Pessimistic time, and t_n = Most likely time)
(a) $\frac{t_o+t_p+t_n}{3}$ (b) $\frac{t_o+2t_p+t_n}{4}$ (c) $\frac{t_o+4t_p+t_n}{5}$ (d) $\frac{t_o+t_p+4t_n}{6}$

(iv) For a single channel service centre, if λ is greater than μ,
(a) the queue will be zero
(b) the queue will be infinite
(c) queue will be finite when the service started
(d) the queue will remain same as that.

(v) A change in the objective function for a non-basic variable can affect
(a) $C_j - Z_j$ values of all non-basic variables
(b) $C_j - Z_j$ values of all basic variables
(c) only the $C_j - Z_j$ value of that variable
(d) none of the above.

(vi) The solution to a transportation problem with m-rows (supplies) and n-columns (destinations) is feasible if number of positive allocations is
(a) m+n (b) mn (c) m+n-1 (d) m+n+1.

(vii) If there were n workers and n jobs, there would be
(a) n! solutions (b) (n-1)! Solutions
(c) $(n!)^n$ solutions (d) n solutions.

(viii) A pessimistic decision making criterion is
(a) maximax (b) equally likely
(c) maximin (d) decision making under certainty.

(ix) If an activity has zero slack, it implies that
(a) it lies on the critical path (b) it is a dummy activity
(c) the project progressing well (a) none of the above.

(x) In non-linear programming problem
(a) a local minimum of a strictly convex function on a convex set is a unique global minimum of that function
(b) a local maximum of a concave function on a convex set is not a global maximum of that function
(c) if the constraints of an NLPP are equations optimum solution cannot be obtained by using Lagrange multipliers
(d) none of the above.

Group – B

2. (a) A company has 3 alternatives open, each of which can be followed by any of the four possible events. The conditional pay offs (in Rs.) for each action event combination are given below;

Alternatives	Payoffs conditional on events			
	A	B	C	D
X	8	0	-10	6
Y	-4	12	18	-2
Z	14	6	0	8

Determine which alternative should the company choose, if they adopt the
(a) maximin criterion (b) maximax criterion (c) Hurwicz criterion (α = 0.70) (d) Laplace criterion (e) minimax regret criterion.

(b) A project schedule has the following data.

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time(Days)	4	1	1	1	6	5	4	8	1	2	5	7

From the above information, do the following,
(i) Construct a network diagram.

(ii) Compute earliest event and latest event time.
(iii) Find the total float for each activity.
(iv) Find the critical path and total project duration

6 + 6 = 12

3. (a) The demand for a seasonal product is given below

Demand during season	40	45	50	55	60	65
Probability	0.10	0.20	0.30	0.25	0.10	0.05

The product costs Rs. 60/- per unit and sells at Rs. 80/- per unit. If the units are not sold within the season, they will have no market value. Determine the optimum number of units to be produced.

(b) A small project is composed of seven activities whose time estimates are listed in the table below:

Activity	Estimated duration in weeks		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) Draw the network
(ii) Find the expected duration and variance of each activity
(iii) Show the critical path and determine the project duration

6 + 6 = 12

Group – C

4. (a) Use Vogel's Approximation Method to obtain an initial feasible solution of the following Transportation Problem:

	D	E	F	G	AVAILABLE
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
REQUIREMENT	200	225	275	250	

(b) The Head of the Department has five jobs A,B,C,D,E and five subordinates V, W, X, Y, Z. The number of hours each man would take to perform each job is as follows:

	V	W	X	Y	Z
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

How would the jobs be allocated to minimize the total cost?

6 + 6 = 12

5. (a) Solve the following LPP by Simplex Method :
Maximize; $Z= 10X_1 + 5X_2$
Subject to the constraints: $4X_1 + 5X_2 \leq 100$
$5X_1 + 2X_2 \leq 80$
$X_1 \geq 0, X_2 \geq 0$

(b) A company produces 2 types of hats A and B. Every hat A requires twice as much production time as the second hat B. If the company produces only hat B then it can produce a total of 500 hats per day. The market limits daily sales of hat A and B to 150 and 250 respectively. The profits on hat A and B are Rs.8 and Rs.5 respectively. Solve graphically to get the optimal solution.

6 + 6 = 12

Group – D

6. (a) State the cause of formation of a queue. Identify and discuss the components of a queuing system.

(b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day following a Poisson distribution. Assuming that service time (the time taken to hump a train) distribution follows an exponential distribution with an average of 36 minutes, calculate the following.
(i) The expected queue size.
(ii) The probability that queue size exceeds 10.
If the input of the train increases to an average of 33 per day, what will be the changes in (i) and (ii)?

(1 + 3) + (2 + 2 + 4) = 12

7. (a) Customers arrive at a booking office window being manned by a single individual at a rate of 25 per hour. The time required to serve a

customer has exponential distribution with a mean of 120 seconds. Find the average waiting time of a customer.

(b) Customers arrive at a single window drive according to a Poisson Distribution with mean of 10 minutes and service time per customer is exponential with mean of 6 minutes. The space in front of the window can accommodate only three vehicles including the serviced one. Other vehicles have to wait outside this space.
Calculate:
(i) the probability that an arriving customer can drive directly to the space in front of the window
(ii) the probability that an arriving customer will have to wait outside the directed space
(iii) how long an arriving customer is expected to wait before getting the service?

6 + 6 = 12

Group – E

8. (a) Consider the following NLPP;
Min Z= $3X_1^2$-$20X_1$+ $4X_2^2$-$10X_2$+ $2X_3^2$-$15X_3$+$5X_4^2$-$20X_4$+250
By separating the function into four one-variable functions, show that the function is convex. Find the solution to the problem.

(b) For each of the following functions, determine whether the function is convex, concave or neither.
(i) $f(x)=3x_1x_2 -x_1^2-X_2^2$
(ii) $f(x)=5x_1 + 10x_1^2 + 6x_2 + 2x_2^2 - 8x_1 x_2$

6 + 6 = 12

9. (a) In an NLPP if $f(x, y) = x^2 -2xy-4x+2y+2y^2$, calculate the maxima or minima.

(b) The total profit (Z) of a firm depends upon the level of output (Q) and the advertising expenditure (A). Find the maximum profit when
$Z = 800-3Q^2-4Q+2QA-5A^2+48A$

6 + 6 = 12