(b)

Show that the following vectors in $\mathbb{R}^3$ are linearly independent.

$\beta_1=(3,0,4)$, $\beta_2=(-1,0,7)$, $\beta_3=(2,9,11)$.

Apply Gram-Schmidt orthogonalization process on these vectors to obtain three mutually orthogonal vectors.

4 + (3 + 5) = 12

Group – E

8.(a)

A top open ractangular box is to have a volume of 32 cft. Find the dimension of the box so that the total surface area is minimum.

(b)

Find the maxima and minima of the following function:

$f(x,y)=x^3+3xy^2-3y^2-3x^2+4$

6 + 6 =12

9.(a)

Solve the following LPP by using the simplex method:

Maximize $z=3x_1+2x_2$ subject to $x_1+x_2\le 4$

$x_1-x_2\le 2$

$x_1,x_2\ge 0$

(b)

Find the maximum and minimum of $f(x,y)=5x-3y$ subject to the constraint $x^2+y^2=136$

6 + 6 =12

M.TECH/ECE/IT/VLSI/1ST SEM/MATH 5103/2015

2015

Advanced Mathematics
(MATH 5103)

Time Allotted : 3 hrs Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following: 10 x 1=10

(i) If $P(A)=0.30, P(B)=0.25$ and $P(A\cap B)=0.12$ then $P(B|A)$ is
(a) 0.48 (b) 0.40 (c) 0.43 (d) 0.42

(ii) The random variable X follows $Bi(n,p)$. If $E(X)=6$ and $V(X)=4$ then p has value
(a) $1/2$ (b) $1/4$ (c) $2/3$ (d) $1/3$

(iii) A graph G has a spanning tree if and only if G is
(a) regular (b) connected (c) simple (d) tree.

(iv) The eigen values of the matrix $A=\begin{pmatrix}2 & 4\\ 3 & 1\end{pmatrix}$ are
(a) -2,-5 (b) 2,-5 (c) -2,5, (d) 2,5.

(v) Which of the following is not a linear transformation from $\Re^3$ to $\Re^3$?
(a) $T(x,y,z)=(x,2y,3x-y)$ (b) $T(x,y,z)=(2x,2y,5z)$
(c) $T(x,y,z)=(-x+y+z,x-y+z,x+y-z,x+y+z)$ (d) $T(x,y,z)=(x+1,y+1,z+1)$.

(vi) Euler's formula for planar graphs states that (with standard notations)
(a) $f=e-n+1$ (b) $e=f-n+1$ (c) $n=e+f+2$ (d) $f=e-n+2$

(vii) To colour the vertices of a tree the minimum number of colours required is
(a) 1 (b) 2 (c) 3 (d) 4

(viii) T is a linear transformation from a 10 dimensional vector space into a 20 dimensional vector space with the dimension of its kernel being 5. Then the dimension of its image space will be:
(a) 10 (b) 15 (c) 0 (d) 5

(ix) If the feasible set of an optimization problem is unbounded, then
(a) no finite optimum point exists
(b) it has an infinite number of feasible points
(c) the existence of a finite optimum point cannot be assured
(d) None of these.

(x) The number of stationary points of the function $f(x) = x^2 + 3y$ is
(a) 1 (b) 2 (c) 3 (d) none of these

Group – B

2. The probability density function is given by

$$f(x) = \begin{cases} kx^2, 0 \le x \le 6; \\ k(12-x)^2, 6 \le x \le 12; \\ 0, elsewhere. \end{cases}$$

(i) Evaluate the constant k
(ii) Find $P(6 \le x \le 9)$.

(6 + 6) = 12

3.(a) A Markov chain $\{X_0, X_1, X_2, ...\}$ with states 0,1,2 has the transition probability matrix:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

If $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{4}$ and $P(X_0 = 2) = \frac{1}{2}$, find $E(X_2)$.

(b) Two white balls and two black balls are distributed into two urns so that each urn contains two balls. Then one ball is randomly selected from each urn and their urns are interchanged (the ball selected from the first urn goes to the second urn and vice versa). This process of selecting balls and interchanging urns is repeated many times. Let X_n denote the number of white balls in the first urn after repeating this process n times. What is the state space of this Markov chain? Find out the underlying transition probability matrix.

8 + (1 + 3) =12

Group – C

4.(a) Let G be a connected graph. Prove that G is an Euler graph iff degree of every point of G is even.

(b) State Hall's Marriage Theorem. Show that the following collection of sets satisfies the marriage condition.
$S = \{A_1, A_2, A_3\}$ where $A_1 = \{1,2,3,4\}$, $A_2 = \{1,5,6,7\}$, $A_3 = \{3,4,6\}$.
Produce a traversal (system of distinct representatives) for this collection S.

4 + (2 + 4 + 2) = 12

5.(a) Let G be a simple connected planar graph with n vertices, e edges and f regions. Apply Euler's formula for planar graphs to show that $e \le 3n - 6$.

(b) State Decomposition theorem for chromatic polynomials. Draw a rectangle with one diagonal. Consider a graph by treating the four vertices of the rectange as vertices of the graph and the lines as edges of the graph. Find its chromatic polynomial by applying decomposition theorem.

6 + (1+5) =12

Group – D

6.(a) Let T be a function from $\mathbb{R}^3$ into $\mathbb{R}^3$ defined by
$T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, x_1 + 3x_2 + 2x_3, 2x_1 + 5x_2 + 5x_3)$.
(i) Show that T is a linear transformation
(ii) Find the dimension of the kernel of T

(b) Define an inner product over a vector space.Determine whether the vectors (1,0,1), (0,-5,0) and (-1,0,1) form an orthogonal basis for the Euclidean space $\mathbb{R}^3$ under the standard inner product.

(3 + 3) + (2 + 4) = 12

7.(a) Find out the eigen values of the following matrix:

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}.$$