INFORMATION THEORY & CODING
(INFO 2203)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If X is the message code vector and H is the parity check matrix then value of XH^T is always_____.
(a) 1 (b) 0
(c) dependent on H (d) none of these.

(ii) For a (2, 1, 2) convolution encoder, impulse response is 1101. The output code word for data 101 is _____.
(a) 01110111 (b) 11011101
(c) 11001110 (d) 01010101.

(iii) If $I(x_1)$ and $I(x_2)$ are the information carried by two symbols x_1 and x_2 respectively then information carried compositely by x_1 and x_2 is _____.
(a) $I(x_1, x_2) = I(x_1) + I(x_2)$ (b) $I(x_1, x_2) = I(x_1) - I(x_2)$
(c) $I(x_1, x_2) = I(x_1) * I(x_2)$ (d) none of these.

(iv) A DMS with two symbols x_1, x_2 and $P(x_1) = 0{\cdot}1$ and $P(x_2) = 0{\cdot}9$ are coded as 0 and 1 respectively. Thus the entropy is_____.
(a) 0·469 b/symbol (b) 0·569 b/symbol
(c) 0·369 b/symbol (d) 0·669 b/symbol.

(v) For GF (2^3) the elements in the set are:
(a) { 1, 2, 3, 4, 5, 6, 7 } (b) { 0,1, 2, 3, 4, 5, 6 }
(c) { 0, 1, 2, 3 } (d) { 0, 1, 2, 3, 4, 5, 6, 7 }.

(vi) The coding efficiency is expressed as _____.
(a) 1 – redundancy (b) 1 + redundancy
(c) 1/redundancy (b) none of these.

(vii) The syndrome polynomial in a cyclic code that solely depends on
(a) generator polynomial (b) parity polynomial
(c) error polynomial (d) code word.

(viii) Basically, Galois field consists of _____ number of elements.
(a) finite (b) infinite
(c) both (a) and (b) (d) none of the above.

(ix) Golay code is _____.
(a) (27, 13) code (b) (25, 13) code
(c) (27, 12) code (d) (25, 12) code.

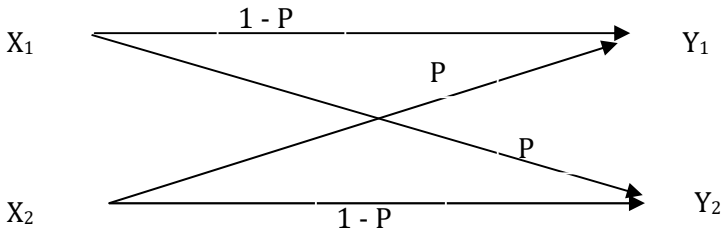
(x) The minimum distance (d_{min}) of a code is 5. How many errors it can detect and correct?
(a) 3 (b) 2 (c) 4 (d) 1.

Group – B

2. Consider a source that, emits nine symbols with the following probabilities – 0.25, 0.15, 0.15, 0.12, 0.10, 0.08, 0.06, 0.05, 0.04.
(i) Find the code word for the symbols using Shannon-Fano algorithm.
(ii) Compare its efficiency with Shannon-Fano-Elias algorithm.

(5 + 1) + (5 + 1) = 12

3. (a) Consider a BSC in figure below with $P(X_1) = k$.



(i) Prove that the mutual information $I(X; Y) = H(Y) + P \log_2 P + (1 - P) \log_2 (1 - P)$.
(ii) Compute I(X; Y) for k = 0.5 and P = 0.1.

(b) For a channel whose matrix is given below:

$$P(Y \mid X) = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

Find I (X; Y) and channel capacity, given that input symbols occur with equal probability.

(4 + 2) + (3 + 3) = 12

Group – C

4. For a linear block code, the syndrome is given by:
   $S_1 = m_1 + m_2 + m_3 + m_5$
   $S_2 = m_1 + m_2 + m_4 + m_6$
   $S_3 = m_1 + m_3 + m_4 + m_7$
   (i) Find the parity check matrix.
   (ii) Draw the encoder circuit.
   (iii) Find the code words for a few input sequences.
   (iv) How many errors can be detected and corrected?
   (v) What is the syndrome for the received data 1011011?

   2 + 3 + 4 + 2 + 1 = 12

5. The message 100100101 is to be transmitted in a cyclic code with a generator polynomial $g(x) = x^2 + 1$.
   (i) Find the transmitted code word.
   (ii) If the received code word is 10010010100, represent the code in polynomial form, and using CRC determine whether the received code is error free or not.

   6 + 6 = 12

Group – D

6. (a) Construct the field $GF(2^4)$ for the given polynomial $p(x) = x^4 + x + 1$.

   (b) A codeword c(x) of the (15, 5) triple error correcting BCH code incurs errors so as to give $v(x) = x^{13} + x^{10} + x^8 + x^4 + x + 1$. Find the error location polynomial using Reed Solomon Code.

   5 + 7 = 12

7. (a) Find the generator polynomial g(x) for a single error correcting binary BCH code of block length 15 over GF (16). Use primitive polynomial $p(x) = x^5 + x + 1$.

   (b) Find the minimal polynomial for the field element α^3 in GF (2^4). Use the primitive polynomial $p(z) = z^4 + z + 1$ to construct $GF(2^4)$.

   7 + 5 = 12

Group – E

8. Explain Viterbi Algorithm to decode the convolutional codes with suitable example.

   12

9. Design a convolutional encoder of rate ½ with constraint length of 4 and minimum distance of 6.
   (i) Construct the state diagram and trellis diagram of the encoder.
   (ii) What is the generator matrix of the code?

   (4 + 4) + 4 = 12