DESIGN AND ANALYSIS OF ALGORITHMS
(INFO 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is
(a) Queue (b) Stack
(c) Heap (d) B-Tree.

(ii) We use dynamic programming approach when
(a) it provides optimal solution
(b) the solution has optimal substructure
(c) the given problem can be reduced to the 3-SAT problem
(d) it's faster than Greedy.

(iii) $\sum_{i=0}^{n} i^3 = ?$
(a) $\Theta(n^3)$ (b) $\Theta(n^4)$ (c) $\Theta(n)$ (d) $\Theta(3)$.

(iv) Heap sort is found to be very efficient
(a) with regard to storage requirement (b) in time consumption
(c) regarding overheads involved (d) none of the above.

(v) Suppose we need to sort a list of employee records in ascending order, using the social security number (a 9-digit number) as the key (i.e., sort the records by social security number). If we need to guarantee that the running time will be no worse than n log n, which sorting methods could we use?
(a) Mergesort
(b) Quicksort
(c) Either mergesort or quicksort
(d) None of these sorting algorithms guarantee a worst-case performance of n log n or better.

(vi) If every square of the board is visited, then the total number of moves of n-queen problem is
(a) n^3-1 (b) n-1 (c) n^2-1 (d) log n-1.

(vii) A person wants to visit some places. He starts from a vertex and then wants to visit every place connected to this vertex and so on. What algorithm he should use?
(a) Depth First Search (b) Breadth First Search
(c) Prim's algorithm (d) None of the mentioned.

(viii) Ω-Notation provides an asymptotic
(a) upper bound
(b) lower bound
(c) one that is sandwiched between the two bounds
(d) none of these.

(ix) Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?
(a) R is NP-complete (b) R is NP-hard
(c) Q is NP-complete (d) Q is NP-hard.

(x) Recursion is a method in which the solution of a problem depends on
(a) larger instances of different problems
(b) larger instances of the same problem
(c) smaller instances of the same problem
(d) smaller instances of different problems.

Group – B

2. (a) State master's theorem and find the time complexity for the following recurrences:
(i) $T(n) = 4T(n/4) + 1$
(ii) $T(n)= 8T(n/2) + n^2$

(b) Generally, the algorithm for computing x^n takes O(n) time complexity. Write an alternative algorithm to compute x^n which will take O(log n) time complexity.

(c) Define O, Ω and Θ notation.

(2 + 2 + 2) + 3 + 3 = 12

3. (a) Prove that the average case time complexity of quick sort is $O(\log_2 n)$.

(b) Using divide and conquer method solve the following set of unsorted elements using Heap Sort algorithm.

15 19 11 16 18 12 45 39 21

Also analyze the time complexity of the Heap Sort algorithm.

(c) Compare Strassen's Matrix multiplication method with conventional matrix multiplication method.

4 + (3 + 3) + 2 = 12

Group – C

4. (a) Using a permutation tree explain the 4-Queens problem and find out the possible number of solutions.

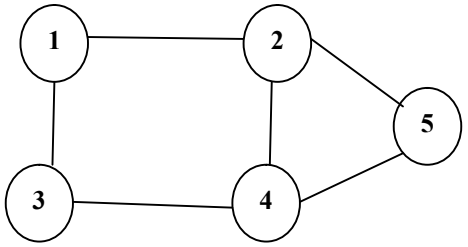
(b) Write an algorithm of N-Queens problem.

(c) What is residual network? Define the residual capacity $C_f(u, v)$ of a flow network.

4 + 4 + (1 + 3) = 12

5. (a) Explain the two different properties of flow graph and define Ford-Fulkerson algorithm.

(b) By considering the following graph show all possible solutions of 4-coloring using a state space tree.



Write an algorithm using backtracking for m-coloring of graph. Hence comment about its time complexity.

(1 + 3) + (3 + 4 + 1) = 12

Group – D

6. (a) Write an algorithm of MATRIX-CHAIN-ORDER (p) and also write a recursive algorithm MATRIX-CHAIN-MULTIPLY (A, s, i, j) that actually performs the optimal matrix-chain multiplication, given the sequence of matrices (A1, A2, ……., An), the s table computed by MATRIX-CHAIN-ORDER, and the indices i and j. (The initial call would be MATRIX-CHAIN-MULTIPLY (A, s, 1, n)).

(b) Compare Prim's and Kruskal's algorithm and deduce time complexity for both.

(5 + 3) + (2 + 2) = 12

7. (a) What is Greedy Method? Write an algorithm of Job Sequencing with deadlines using Greedy Method.

(b) Write down the Floyd Warshall algorithm to find all pair shortest path. Apply the same algorithm to find out all pair shortest path of the graph represented as following adjacency matrix. Also comment on the complexity of the same.

i/j	1	2	3	4
1	0	6	0	2
2	1	0	0	4
3	0	2	0	7
4	1	4	1	0

(1 + 3) + (3 + 4 + 1) = 12

Group – E

8. (a) Explain how you attempt to solve 15-puzzle problem using Branch and Bound strategy. Draw a portion of the state space tree generated by it.

(b) What do you mean by polynomial reductions? Show that clique decision problem is NP complete.

7 + (1 + 4) = 12

9. (a) Describe approximate algorithm for TSP & calculate the performance ratio for that.

(b) Discuss diagrammatically the relations among P class, NP class, NP hard class and NP complete class.

8 + 4 = 12