B.TECH/ BT/ CE/ EE/ ME /4TH SEM/ MATH 2002/2018

NUMERICAL AND STATISTICAL METHODS (MATH 2002)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The order of convergence of bisection method is
(a) linear (b) quadratic (c) cubic (d) 1.5.

(ii) A system of equations AX = b where $A = (a_{ij})_{n\times n}$ is said to be diagonally dominant if
(a) $|a_{ii}| \geq \sum_{j=1, i\#j}^{n} |a_{ij}|$ for all i (b) $|a_{ii}| < \sum_{j=1, i\#j}^{n} |a_{ij}|$ for all i
(c) $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$ for all i (d) $|a_{ii}| < \sum_{j=1}^{n} |a_{ij}|$ for all i.

(iii) In Simpson's one third rule for finding $\int_a^b f(x)dx$, $f(x)$,is approximated by
(a) line segment (b) parabola
(c) circular segment (d) part of ellipse.

(iv) Newton backward interpolation formula is used for
(a) equal intervals (b) unequal intervals
(c) both equal & unequal intervals (d) even no. of intervals.

(v) $\Delta^n x^n = ?$
(a) $n!$ (b) $(n-1)!$ (c) n^2 (d) 0.

(vi) If two events A and B are mutually exclusive, then $P(A \cap B)$ is
(a) –0.01 (b) 0 (c) 1 (d) 0.5.

(vii) An unbiased coin is tossed 4 times. The probability of getting heads exactly 3 times is
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$.

(viii) A random variable X is uniformly distributed in the interval $[a, b]$. Then the mean of X is
(a) $\frac{1}{b-a}$ (b) $\frac{a+b}{2}$ (c) $\frac{b-1}{a-1}$ (d) $\frac{b}{a}$.

(ix) If a Poisson variate X is such that $P(X = 1) = P(X = 2)$, then $P(X = 0)$ is
(a) e^{-1} (b) e^{-4} (c) e^{-2} (d) 1.

(x) The mode and median of the observation 4, 6, 6, 8, 3, 8, 8 & 4 are
(a) 8 and 6 (b) 8.5 and 6.5 (c) 5 and 7 (d) 4 and 3.

Group – B

2. (a) Find the real positive root of the equation $x^3 - 9x + 1 = 0$ by Regula Falsi method correct to three decimal places.

(b) Solve the given system of equations using Gauss Elimination method
$3x + 9y - z = 11$
$4x + 2y + 13z = 24$
$4x - 2y + z = -8$

6 + 6 = 12

3. (a) Solve the following system of equations
$3x_1 + 2x_2 - 4x_3 = 12$
$-x_1 + 5x_2 + 2x_3 = 1$
$2x_1 - 3x_2 + 4x_3 = -3$
by LU factorization method.

(b) Find a positive value of $(17)^{\frac{1}{3}}$ correct upto four decimal places by Newton-Raphson method.

7 + 5 = 12

Group – C

4. (a) Use finite difference method to find the values of a and b in the following table.

x	0	2	4	6	8	10
$f(x)$	−5	a	8	b	20	32

(b) Solve $\frac{dy}{dx}=x^2+y^2$, $y(0)=0$ using Euler's method and find $y(0.1)$ & $y(0.2)$ using h=0.05.

6 + 6 = 12

5. (a) Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ by using Simpson's $\frac{1}{3}$rd rule taking eleven ordinates and hence find the value of $\log_e 2$ correct upto five significant figures.

(b) Find the value of $\sqrt{2}$ using Newton's forward interpolation formula for the given data:

x	1.9	2.1	2.3	2.5	2.7
$f(x)=\sqrt{x}$	1.3784	1.4491	1.5166	1.5811	1.6432

6 + 6 = 12

Group – D

6. (a) A city is partitioned into districts A, B, C having 20%, 40% and 40% of the registered voters respectively. The voters who support party X constitute 50% of the population in district A, 25% in B and 75% in C.
(i) If a registered voter is chosen randomly in the city, find the probability that the voter is a supporter of party X.
(ii) A registered voter of the city is chosen at random and found to be a supporter of party X. Find the probability that the voter came from district B.

(b) 100 prizes will be given in a lottery of 10000 tickets. Find the minimum number of tickets a person has to buy in order that the probability of his wining at least one prize is greater than $\frac{1}{2}$.

6 + 6 = 12

7. (a) Four boxes A, B, C, D contain fuses. The boxes contain 5000, 3000, 2000, and 1000 fuses respectively. The percentages of fuses in the boxes which are defective are 3%, 2%, 1% and 0.5% respectively. One fuse is selected at random arbitrarily from one of the boxes. It is found to be a defective fuse. Find the probability that it has come from box D.

(b) Two newspapers *X* and *Y* are published in a certain city. It is estimated from a survey that 16% read *X*, 14% read *Y* and 5% read both the newspapers. Find the probabilities that a randomly selected person
(i) does not read any newspaper
(ii) read only *Y*.

(c) If *A* and *B* are two independent events, then prove that $\overline{A}$ and $\overline{B}$ are also independent.

6 + 3 + 3 = 12

Group – E

8. (a) Show that the function $f(x)$ given by

$$f(x)=\begin{cases} x, & 0\le x<1 \\ k-x, & 1\le x\le 2 \\ 0, & Otherwise \end{cases}$$

is a probability density function, for a suitable value of the constant *k*. Construct the distribution function of a random variable *X* and compute the probability that the random variable X lies between $\frac{1}{2}$ and $\frac{3}{2}$.

(b) A random variable *X* follows binomial distribution with mean 4 and standard deviation $\sqrt{2}$. Find the probability of assuming the non-zero value of the variable.

(c) If the probability of producing a defective screw is $p=0.01$, what is the probability that a lot of 100 screws will contain more than 2 defectives?

6 + 3 + 3 =12

9. (a) Assuming that the height distribution of a group is normal, find the mean and standard deviation if 84% of the men have heights less than 65.2 inches and 68% have heights lying between 62.8 and 65.2 inches.
[Given $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0.9} e^{-\frac{t^2}{2}}\,dt=0.84$ and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-0.9} e^{-\frac{t^2}{2}}\,dt=0.16$]

(b) For two variables *x* and *y* the equations of two regression lines are $x+2y-5=0$ and $2x+3y-8=0$. Identify which one is the regression line of *y* on *x*. Find the means of *x* & *y*. Find the correlation coefficient between x and y. Estimate σ_y given $\sigma_x=12$.

6 + 6 = 12