B.TECH/ME/4TH SEM/MATH 2001/2018

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of $\oint_C \frac{z^3}{(z-2)^2}dz$, where C is the circle $|z|=1$, is

(a) $24\pi i$ (b) $12\pi i$ (c) $6\pi i$ (d) 0.

(ii) For the function $f(z)=\frac{\sin z}{z}$, $z=0$ is

(a) an essential singularity (b) simple pole
(c) a pole of order 2 (d) a removable singularity.

(iii) If $2x-x^2+my^2$ is harmonic, then $m=$

(a) 0 (b) 1 (c) 2 (d) 3

(iv) Bessel's equation of order zero is

(a) $xy''+y'+xy=0$ (b) $xy''+y'=0$
(c) $xy''-y'+xy=0$ (d) $y'+xy=0$

(v) The solution of $p^2+q^2=n^2$ is

(a) $z=ax\pm\sqrt{n^2-a^2}\,y+c$ (b) $z=ax\pm by$
(c) $z=ax+c$ (d) $z=by+c$

(vi) The Legendre's polynomial of $2x^2+x+3$ is

(a) $\frac{1}{3}[4P_2(x)-3P_1(x)+11P_0(x)]$ (b) $\frac{1}{3}[4P_2(x)+3P_1(x)-11P_0(x)]$
(c) $\frac{1}{3}[4P_2(x)+3P_1(x)+11P_0(x)]$ (d) $\frac{1}{3}[4P_2(x)-3P_1(x)-11P_0(x)]$

(vii) The points of singularities of the ordinary differential equation $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$ are

(a) 1, 0 (b) -1, 0 (c) 1, -1 (d) 0, 2

(viii) A Fourier series is called a half-range series if its expansion contains

(a) both sine and cosine terms (b) only sine or cosine terms
(c) no sine and no cosine terms (d) only constants.

(ix) If the Fourier transform of $f(x)$ be $F(s)$, then the Fourier transform of $f(x+a)$ is

(a) $e^{-ias}F(s)$ (b) e^{-ias} (c) $F(s)$ (d) $\frac{F(s)}{a}$

(x) The order of $\frac{\partial^3 u}{\partial x\partial y^2}=\left(\frac{\partial u}{\partial x}\right)^4$ is

(a) 3 (b) 1 (c) 2 (d) 4.

Group – B

2. (a) Show that the function $u(x,y)=4xy-3x+2$ is harmonic. Construct the corresponding analytic function $f(z)=u(x,y)+iv(x,y)$. Express $f(z)$ in terms of the complex variable z.

(b) Evaluate $\oint_C \frac{e^z dz}{z^2+1}$, where C is the circular path $|z|=2$.

7 + 5 = 12

3. (a) Obtain the Laurent series which represents the function $f(z)=\frac{1}{(1+z^2)(z+2)}$ in $1<|z|<2$.

(b) Evaluate the following integral using the residue theorem:
$\oint_C \frac{4-3z}{z(z-1)(z-2)}dz$, C is the circle $|z|=\frac{3}{2}$.

5 + 7 = 12

Group – C

4. (a) Find the Fourier series to represent $x-x^2$ from $x=-\pi$ to $x=\pi$ and hence find the value of $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\ldots$

(b) If $f(x)=e^{-bx}, b>0, x\geq 0$, find the Fourier cosine transform of $f(x)$ and hence evaluate $\int_0^\infty \frac{\cos sx}{b^2+s^2}ds$.

7 + 5 = 12

5. (a) Express $f(x)=x$ as a half range cosine series in $0<x<2$.

(b) Find the inverse Fourier sine transform of e^{-as}/s, $a>0$.

6 + 6 = 12

Group – D

6. (a) Find the series solution of the ODE $y''+xy'+(x^2+2)y=0$ about the point $x=0$.

(b) State the expression of generating function of Bessel's function. Use this to prove $2nJ_n'(x)=J_{n-1}(x)-J_{n+1}(x)$.

7 + (1 + 4) = 12

7. (a) Prove $\int_{-1}^{1}[P_n(x)]^2\,dx=\frac{2}{2n+1}$

(b) Solve by Finite Difference method $y''(x)+y(x)+1=0$ with given boundary condition $y(0)=0, y(1)=0$ and take $h=0.5$.

6 + 6 = 12

Group – E

8. (a) Form the partial differential equation by eliminating the arbitrary functions from $z=f(x+at)+g(a-xt)$.

(b) Solve $\frac{y^2z}{x}p+xzq=y^2$.

6 + 6 = 12

9. (a) Solve $4\frac{\partial^2 z}{\partial x^2}+12\frac{\partial^2 z}{\partial x\partial y}+9\frac{\partial^2 z}{\partial y^2}=e^{3x-2y}$.

(b) Solve by the method of separation of variables $\frac{\partial u}{\partial x}+u=\frac{\partial u}{\partial t}$, $u=4e^{-3x}$ at $t=0$.

6 + 6 = 12