

Group - D

6. (a) When is a Type 2 (context-free) grammar said to be ambiguous?
Consider the grammar $G = (V, T, P, S)$, where $V = \{S, A, B, 0, 1, 2, 3, 4\}$, $T = \{0, 1\}$, S is the start symbol, and the productions are as follows:
 $S \rightarrow A, S \rightarrow B, S \rightarrow 0, S \rightarrow 1, A \rightarrow 01S1S, B \rightarrow 01S1S1S$.
Show that the grammar G is ambiguous.
- (b) Design a pushdown acceptor that will recognize the following context-free language L over the input alphabet $\{0, 1\}$:
 $L = \{\alpha \mid \text{the string } \alpha \text{ has an equal number of 0's and 1's}\}$.
- 6 + 6 = 12
7. (a) Consider the language $L_{12} = \{0^m 1^n 0^n 1^m \mid m, n > 0\}$. Provide a context-free grammar for L_{12} thereby showing that L_{12} is a context-free language.
- (b) Consider the following context-free grammar G :
- $S \rightarrow A$
 $S \rightarrow \epsilon$
 $A \rightarrow BAB$
 $A \rightarrow BBB$
 $B \rightarrow 00C$
 $B \rightarrow 1$
 $C \rightarrow 1$
- Convert G to Chomsky Normal Form.

6 + 6 = 12

Group - E

- 8 Two positive integers m and n are written on a semi-infinite Turing machine tape in unary notation. The integer m is to the left of the integer n on the tape, a blank cell precedes the first 1 of m , and a single blank cell separates the two integers. At start the read/write head is positioned on the leftmost 1 of m . Give the state transition diagram of a Turing machine that will halt on the leftmost 1 of m if $m = n$, and will halt on the leftmost 1 of n if $m \neq n$. Clearly state any assumptions made.
- 12
9. (a) Given two positive integers x and y , design a Turing Machine that computes $(x+y)$.
- (b) What is the 'Halting Problem' for Turing Machines? What do we mean when we say that the Halting Problem for Turing machines is unsolvable?
- 6 + (3 + 3) = 12

**THEORY OF COMPUTATION
(CSEN 5201)**

Time Allotted : 3 hrs

Full Marks : 70

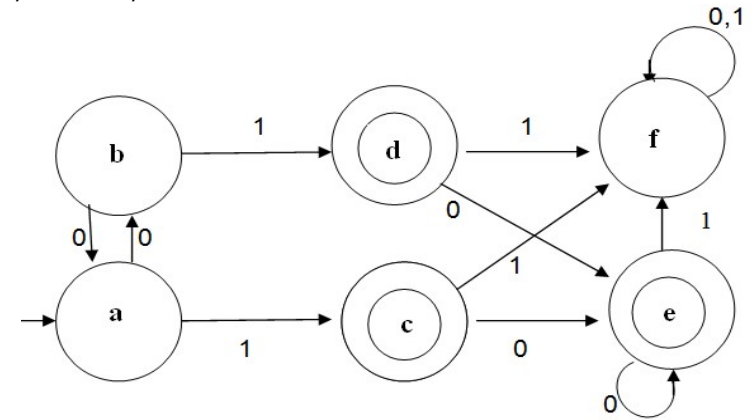
*Figures out of the right margin indicate full marks.**Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.**Candidates are required to give answer in their own words as far as practicable.***Group - A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) Consider the language $L = \{0^n 1^n 0^m 1^m \mid n > 0, m > 0\}$ on the input alphabet $\{0, 1\}$. Which one of the following statements is true?
 (a) L can be recognized by a suitably designed deterministic finite state acceptor (dfsa).
 (b) L can be recognized by a suitably designed deterministic pushdown acceptor (dpda) but not by any dfsa.
 (c) L can be recognized by a suitably designed non-deterministic pushdown acceptor (ndpda) but not by any dpda.
 (d) L can be recognized by a suitably designed Turing Machine but not by any ndpda.
- (ii) Which of the languages will be accepted by a deterministic finite state automaton (DFSA)?
 (a) $L_1 = \{a^n b^n : n > 1\}$ (b) $L_2 = \{a^n : n > 10\}$
 (c) $L_3 = \{a^p : p \text{ is any prime number}\}$ (d) $L_4 = \{a^n b^n c^n : 10 \geq n \geq 0\}$.
- (iii) L_1 and L_2 are two regular (Type 3) languages on a given input alphabet. Let the language L consist of all strings that are neither in L_1 nor in L_2 . Then
 (a) L is a regular language.
 (b) L is a context-free language but not necessarily a regular language.
 (c) L is a context-sensitive language but not necessarily a context-free language.
 (d) L is a Type 0 language but not necessarily a context-sensitive language.
- (iv) How many distinct arrangements of length 5 of the digits $\{0, 1, 2, 3\}$ with repetitions allowed contain exactly two 3's?
 (a) 128. (b) 135. (c) 256. (d) 270.

- (v) Which one of the following languages cannot be accepted by a non-deterministic pushdown acceptor (ndpda)?
 (a) $\{ 0^m 1^n \mid 0 < m, 0 < n \}$ (b) $\{ 0^m 1^n 0^m \mid 0 < m, 0 < n \}$
 (c) $\{ 0^m 1^n 0^m \mid 0 < m \}$ (d) $\{ 0^m 1^n 0^r \mid 0 < m, 0 < n, 0 < r \}$.
- (vi) Let n be a positive integer that satisfies the condition $(17000 \leq n \leq 32000)$. When n is written in octal notation with no leading zeroes, p octal digits are needed. When n is written in hexadecimal notation with no leading zeroes, q hexadecimal digits are needed. Then which one of the following statements is true?
 (a) $p = 7, q = 5$ (b) $p = 6, q = 5$ (c) $p = 6, q = 4$ (d) $p = 5, q = 4$.
- (vii) If a language L_6 is regular then which statement is FALSE?
 (a) There exists a Pushdown automaton which accepts L_6 .
 (b) There exists a Turing Machine which accepts L_6 .
 (c) There exists a Nondeterministic finite automaton which accepts L_6 .
 (d) None of the above statements are correct.
- (viii) If languages of type X are closed under union and complementation operations then which of the following is correct?
 (a) Type X languages must be closed under intersection.
 (b) Type X languages may not be closed under intersection
 (c) Type X languages will never be closed under intersection.
 (d) Cannot say anything about the intersection property of Type X languages.
- (ix) Let the input alphabet be $\{ 0,1,2 \}$, and let the language L consist of all strings on the input alphabet that contain at least one 1 and at least one 2, no condition being imposed on the number of 0's in a string. Then the deterministic finite state acceptor (dfsfa) with fewest states that recognizes L has n states where n equals
 (a) 4 (b) 5 (c) 6 (d) 7.
- (x) Which of the following identities for regular expression is FALSE?
 [Here P, Q, R are all regular expressions, ϕ denotes null set and ϵ denotes empty string]
 (a) $R+R=R$ (b) $(PQ)^*P=P(QP)^*$ (c) $\phi^* = \epsilon$ (d) $(P+Q)^*=(PQ)^*$.

Group - B

2. (a) Let L_{10} be the set of all palindromes over $\{a, b\}$. Construct a grammar G generating L_{10} . According to Chomsky classification, which type of grammar is this?
 (b) A finite state machine M_1 has the state transition diagram shown below. The start state is **a**, and the final states are **c**, **d** and **e**. Minimize the number of states in the machine.



(3 + 2) + 7 = 12

3. Each of the claims given below is either TRUE or FALSE. For each claim, clearly state TRUE or FALSE, briefly giving reasons and examples in support of your choice.
- (i) Claim: A deterministic finite state acceptor (dfsfa) M must have a dead state, i.e., a non-goal state having no outgoing transitions to other states.
 - (ii) Claim: A non-deterministic finite state acceptor (ndfsa) N can have more than one start state.
 - (iii) Claim: If there is an ϵ -transition from a state X to another state Y in a non-deterministic finite state acceptor (ndfsa) N , then when N is converted to a reduced deterministic finite state acceptor (dfsfa) M , the states X and Y will not remain separate states but will coalesce into one state.

(4 + 4 + 4) = 12

Group - C

4. (a) Construct a deterministic finite state acceptor (dfsfa) M on the input alphabet $\{ 0,1 \}$ that accepts a string α if and only if α is contained in the regular expression $(00+11)^*0^*$.
 (b) Construct a deterministic finite state acceptor (dfsfa) N on the input alphabet $\{ 0,1 \}$ that accepts a string α if and only if α is *not* contained in the regular expression $(00+11)^*0^*$.
5. (a) Use the Pumping Lemma for Regular Languages to show that the language $L_{11} = \{a^{n!} : n \text{ is any integer } > 3\}$ is not regular.
 (b) Prove that the family of Regular Languages is closed under intersection and concatenation operation.

8 + 4 = 12

6 + (3 + 3) = 12