B.TECH/EE/6TH SEM/ELEC 3231/2018

DIGITAL SIGNAL PROCESSING (ELEC 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The fundamental period (N) of a periodic signal $x(n) = 3\sin\left(\frac{16\pi}{57}\right)$ is

(a) N = 29.5 (b) N = 6 (c) N = 18 (d) N = 57.

(ii) The even component of the signal $x(n) = u(n)$ is

(a) $x_e(n) = \begin{cases} -0.5, & for\ n > 0 \\ 1, & for\ n = 0 \\ 0.5, & for\ n < 0 \end{cases}$

(b) $x_e(n) = \begin{cases} 0.5, & for\ n > 0 \\ 0, & for\ n = 0 \\ -0.5, & for\ n < 0 \end{cases}$

(c) $x_e(n) = \begin{cases} 0.8, & for\ n > 0 \\ 1, & for\ n = 0 \\ 0.8, & for\ n < 0 \end{cases}$

(d) $x_e(n) = \begin{cases} 0.5, & for\ n > 0 \\ 1, & for\ n = 0 \\ 0.5, & for\ n < 0 \end{cases}$

(iii) From a given signal $x(n)$, a signal $g(n) = x\left(\frac{2n}{3}\right)$ can be generated using

(a) up-sampling and down-sampling
(b) down-sampling and up-sampling
(c) up-sampling, delay, and down-sampling
(d) down-sampling, delay, up-sampling.

(iv) The length of the output response $y(n)$ with impulse response $h(n) = (2,\ 1,\ 1,\ 1),\ 0 \leq n \leq 3$ to the input signal $x(n) = (1,\ 2), 0 \leq n \leq 1$ is

(a) 6 (b) 8 (c) 7 (d) 5.

(v) The R.O.C of $z - transform$ for the discrete signal $x(n) = 3^n u(-n)$ (non-causal signal) is

(a) R.O.C: $|z| < 3$
(b) R.O.C: $|z| > 3$
(c) R.O.C: Complete $z - complex$ plane
(d) R.O.C: Inside the unit circle of $z - plane$.

(vi) The impulse response of a discrete causal system is given as $h(n) = a^n u(n)$, the system is bounded-input-bounded-output stable if

(a) The impulse response is absolutely square summable and $|a|$>1.
(b) The impulse response is absolutely square summable and $|a| < 1$.
(c) The impulse response is absolutely summable and $|a|$>1.
(d) The impulse response is absolutely summable and $|a|$ <1.

(vii) If the DFT of four point segment signal $x(n)$ is $X(0) = 4,\ X(1) = -j2,\ X(2) = 0,\ X(3) = j2$, then DFT of the corresponding time reversed signal $g(n) = x(-n)$ is

(a) $X(0) = 4,\ X(1) = j2,\ X(2) = 0,\ X(3) = -j2$
(b) $X(0) = 4,\ X(1) = j2,\ X(2) = 0,\ X(3) = -j2$
(c) $X(0) = 4,\ X(1) = -j2,\ X(2) = 0,\ X(3) = j2$
(d) $X(0) = 0,\ X(1) = j2,\ X(2) = 4,\ X(3) = -j2$.

(viii) A signal $x(n)$ as a 4-point sequence and $X(k) = \{9,\ b\ ,\ 1,\ -1-j2\}$ (for $k = 0\ to\ 3$) as its 4-point DFT. The $DFT\ point$ at $k = 2,\ i.e.$ the value of $'b'$ is

(a) $-1 + j2$ (b) $\sqrt{5}$ (c) $-2 + j1$ (d) $1 - j2$

(ix) Let $x(n) = \{1,\ 2,\ 0,\ 3\}$ for $n = 0\ to\ 3$. The circularly shifted signal $x(n+2)$ is

(a) $\{0,\ 3,\ 1,\ 2\}$ for $n = 0\ to\ 3$
(b) $\{1,\ 3,\ 0,\ 2\}$ for $n = 0\ to\ 3$
(c) $\{3,\ 0,\ 1,\ 2\}$ for $n = 0\ to\ 3$
(d) $\{2,\ 0,\ 3,\ 1\}$ for $n = 0\ to\ 3$.

(x) In a linear digital system, if P_x is the power of the input signal and P_y is the power output signal, then they are related by

(a) $P_y = 0.9P_x$ (b) $P_y = H(e^{j\Omega})P_x$
(c) $P_y = H^2(e^{j\Omega})P_x$ (d) $P_y = |H(e^{j\Omega})|^2 P_x$

Group – B

2. (a) (i) Show that $\sum_{n=-M}^{M} x(n) = x(0) + 2\sum_{n=1}^{M} x(n)$ if $x(n)$ is even and $\sum_{n=-M}^{M} x(n) = 0$ if $x(n)$ is odd.

(ii) Check the discrete time system $y(n) = x(nM)$ (i.e. down-sampler is considered as a system) for linearity and time invariance.

(b) (i) Determine the impulse response of the system $y(n) = a(0)x(n) + b(1)y(n-1)$; where $y(-1) = 0$; and establish the condition for which the system is stable. Is the system causal?

(ii) Realize the following digital filter using direct-form II structure:

$$H(z) = \frac{(0.7157z^2 + 1.4314z + 0.7151)}{(z^2 + 1.349z + 0.514)}$$

(2 + 2) + (4 + 4) = 12

3. (a) (i) A linear time invariant system with impulse response $h(n)$is stable, in the bounded-input bounded-output sense, if and only if the impulse response is absolutely summable, that is if $\sum_{n=-\infty}^{\infty}|h(n)| < \infty$.

(ii) If $x(n)$ is the input and $h(n)$ is the impulse response of a linear time-invariant discrete system $'S'$ then output $y(n)$ is given by $y(n) = \sum_{m=-\infty}^{\infty} x(m)h(n-m)$.

(b) Find the linear convolution of the sequences $x(n) = \{1, 2, 3, 2\}; for\ -4 \le n \le -1$ and $h(n) = \{1, 4, -2, 3\}; for\ 1 \le n \le 4$ as an output of the system $(i.e.\ y(n) = x(n) * h(n))$ using **numerical computation** (vector-matrix computation).

(4 + 3) + 5 = 12

Group – C

4. (a) Find the $z-$ transform of the signal $x(n) = \left[3\left(\frac{4}{5}\right)^n - \left(\frac{2}{3}\right)^{2n}\right]u(n)$ (causal signal) and its R.O.C.

(b) An IIR filter (system) is described by the difference equation $y(n) - y(n-1) - 2y(n-2) = x(n)$; with $x(n) = 6u(n)$.
Find (i) the transfer function $H(z) = \frac{Y(z)}{X(z)}$ of the system (assuming initial conditions are zero) and hence obtain the poles and zeros (ii) the forced, natural response, total response, transient response and steady-state response of the system considering the following initial conditions $y(-1) = -1,\ y(-2) = 4$.

5 + 7 = 12

5. (a) Find the inverse $z-$transform of $X(z) = \frac{z}{(z-1/2)\left(z-\frac{1}{3}\right)}$.

(b) Transform $H(s) = \frac{s+1}{(s^2+5s+6)}$ into a digital filter $(H(z))$ using the bilinear transformation. Choose sampling period $T = 1sec$. Express the system as a difference equation suitable for computer processing.

6 + 6 = 12

Group – D

6. (a) Define the DFT of a finite sequence and state the fundamental assumptions that are essential to derive the DFT expressions.

(b) Given a sequence $x(n) = \{1,\ 2,\ 3\ ,4\}$ for $0 \le n \le 3$, evaluate its DFT coefficients $X(k)$, and then compute the amplitude spectrum, Phase spectrum and power spectrum for $X(3)$ only. Highlight the significance of DFT coefficients in context with the signal $\boldsymbol{x(n)}$.

4 + 8 = 12

7. (a) Show that DFT of $\{x(n)\}$ with N-data points can be computed efficiently as the sum of DFT of a signal $a(n) = \left\{x(n) + x\left(n + \frac{N}{2}\right)\right\}$ with $\frac{N}{2}$ data points and DFT of a signal $b(n) = \left\{x(n) - x\left(n + \frac{N}{2}\right)\right\}$ with $\frac{N}{2}$ data points.

(b) Given the DFT sequence $X(k) = \{\ 10,\ -2 + j2,\ \ -2,\ -2 - j2\}$ for $0 \le k \le\le 3$, evaluate its inverse DFT $x(n)$ using decimation-in-frequency FFT.

6 + 6 = 12

Group – E

8. (a) Discuss briefly the frequency response of a linear digital system $H(z)$ when the system is excited with a causal sinusoidal input signal $x(n) = Asin(\Omega n)u(n)$. Derive the steady-state output response of the system as $y(n) = A|H(\Omega)|\ sin(n\Omega + < H(\Omega))$.

(b) What is an all-pass filter? Explain. What form does the transfer function of such filter have? Show the poles and zeros distribution in $z - plane$. Comments on the stability of the system.

6 + 6 = 12

9. (a) With a suitable example, state and explain the design specification of a "lowpass-filter".

(b) Give a brief outline how one can design low-pass FIR filter using *''Hamming window''* technique. Explain each step clearly.

4 + 8 = 12