B.TECH/CHE/6TH SEM/CHEN 3232/2018

COMPUTATIONAL FLUID DYNAMICS
(CHEN 3232)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In hybrid scheme to find the value of property ϕ at face, central difference scheme is applied for
(a) Pe=2 (b) Pe>2 (c) Pe<2 (d) 2<Pe<10.

(ii) In a multigrid iteration scheme, the internode distance at level 3 of coarse grid is equal to ______________, when 'h' is the internode distance for fine grids.
(a) 2h (b) 8h (c)4h (d) 16h.

(iii) $\left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + u^2 = 0$ is ____________ order quasilinear equation
(a) 2nd (b) 3rd (c)1st (d) 0th.

(iv) In SIMPLE algorithm it is
(a) assumed that the errors at all the neighbourhood nodes are zero
(b) assumed that the summation of errors at all the neighbourhood nodes are zero
(c) assumed that the summation of error at the evaluating node is zero
(d) assumed that the error in the convective term is equal to error in pressure.

(v) Neuman boundary condition in heat transfer problem
(a) applies when the boundary temperature is specified
(b) applies when the boundary temperature is held at zero
(c) applies when the heat flow through the boundary is specified
(d) applies when both the boundary temperature and the heat flow through the boundary are specified.

(vi) For a creeping flow, Navier-Stoke's equation will be reduced to
(a) $\frac{Du}{Dt} = \mu\nabla^2 u$ (b) $\frac{Du}{Dt} = -\nabla p$
(c) $-\nabla p + \mu\nabla^2 u = 0$ (d) $\nabla p = 0$.

(vii) In a marching problem the differential equation is
(a) parabolic (b) hyperbolic
(c) elliptic (d) normal ODE.

(viii) For irrotational flow
(a) ∇.u=0 (b) u(∇.u)=0 (c)u(∇xu)=0 (d) ∇xu=0.

(ix) In power law scheme to solve the flow domain, the value of property ϕ at face can be said as ____________________ for 0<Pe<10 and u<0

(a) $\phi_{i-\frac{1}{2}} = \phi_{i-1} - \frac{\left(1-0.1Pe_{i-\frac{1}{2}}\right)^5}{Pe_{i-\frac{1}{2}}}\left[\phi_{i-1} - \phi_i\right]$ (b) $\phi_{i+\frac{1}{2}} = \phi_{i+1} - \frac{\left(1-0.1Pe_{i+\frac{1}{2}}\right)^5}{Pe_{i+\frac{1}{2}}}\left[\phi_i - \phi_{i+1}\right]$

(c) $\phi_{i+\frac{1}{2}} = \phi_i - \frac{\left(1-0.1Pe_{i+\frac{1}{2}}\right)^5}{Pe_{i+\frac{1}{2}}}\left[\phi_{i+1} - \phi_i\right]$ (d) $\phi_{i-\frac{1}{2}} = \phi_{i-1} - \frac{\left(1-0.1Pe_{i-\frac{1}{2}}\right)^5}{Pe_{i-\frac{1}{2}}}\left[\phi_i - \phi_{i-1}\right]$

(x) Finite volume scheme relies on
(a) the conservation form of the balance equations
(b) non-divergence form of the balance equations
(c) the continuity equation alone
(d) none of the above.

Group – B

2. (a) Show that the equation $\frac{\partial^2 u}{\partial t^2} - \beta\frac{\partial^2 u}{\partial x^2} + u = 0$ is hyperbolic, when β is positive.

(b) The governing equation of motion for one-dimensional, inviscid flow is given by the Euler equation. If the system of perfect gas is imposed, the system is written as

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho a^2\frac{\partial u}{\partial x} = 0$$

Classify the system based on eigen value.

4 + 8 = 12

3. (a) Show that the 1-D Euler equation can be written in terms of the primitive variables R=[ρ,u,p]^T as follows $\frac{\partial R}{\partial t} + M\frac{\partial R}{\partial x} = 0$, where $M = \begin{bmatrix} u & \rho & 0 \\ 0 & u & \rho^{-1} \\ 0 & \rho a^2 & u \end{bmatrix}$.

Assume for an ideal gas $p = (\gamma - 1)\left(E - \frac{\rho u^2}{2}\right)$; $E = \rho e + \frac{\rho u^2}{2}$, where e=internal energy, ρ=density of the fluid and u=fluid velocity. **12**

(b) "In the integral form of the transport equation for property ϕ, a term $\int_A n\cdot(\Gamma \,\text{grad}\phi)\, dA$" represents net increase of the property due to inflow diffusional flux across the surface of the control volume". Elaborate the correctness of the statement.

8 + 4 = 12

Group – C

4.(a) Diagrammatically show the computational molecule for a 2D and 3D finite difference scheme.

(b) Write out the finite difference form of the mixed derivative $\frac{\partial^2 \phi}{\partial x \partial y}$ using the central difference scheme in a 2D domain.

8 + 4 = 12

5. Heat is flowing in a rectangular slab of metal and can be modelled using the equation $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ at steady state. The left edge of the slab is maintained at T_H and the right edge is at T_C. All other [illegible] are maintained at temperature T_{amb}. Write down the general form of the [illegible] equation after applying finite difference scheme to solve temperat[illegible] the form $AT = Q$.

Assuming 4 grid points in the x and y direction of the slab, sh[illegible] the exact form of the A matrix.

6 + 6 = 12

Group – D

6. Source-free heat conduction occurs inside an insulated 0.5 m rod whose ends are maintained at constant temperatures of 100°C and 500°C respectively. The one-dimensional heat problem is governed by $\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$. Calculate the steady state temperature distribution in the rod. Thermal conductivity k=1000 W/mK and cross sectional area=10 x 10^{-3} m^2. Consider at least 5 nodes inside the rod and show atleast two iterations. **12**

7. In a steady 2-D situation, the variable ϕ is governed by the equation,
div(ρuϕ) = div(Γgrad ϕ) + a – bϕ, where ρ=1, Γ=1, a=10 and b=2.
The flow field is such that u=1 and v=4 everywhere.
For the uniform grid shown in figure 1, find out the values for ϕ$_1$, ϕ$_2$, ϕ$_3$ and ϕ$_4$ using upwind interpolation scheme.

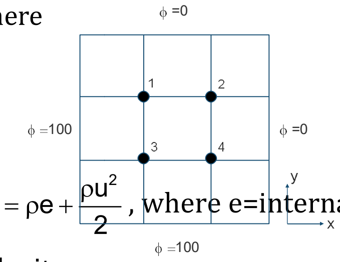

Figure 1

12

Group – E

8. A 1D flow through a porous material is governed by c|u|u+dp/dx=0, where c is a constant. The continuity equation is d(uA)/dx=0, where A is the effective area for the flow. Use SIMPLE algorithm for grid shown in the figure 2 to calculate p$_2$, u$_B$ and u$_C$ from the following data.

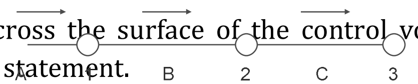

Figure 2

$x_2 - x_1 = x_3 - x_2 = 2$; $c_B = 0.25$; $c_C = 0.2$; $A_B = 5$; $A_C = 4$; $p_1 = 200$; $p_3 = 38$.
As an initial guess set $u_B = u_C = 15$ and $p_2 = 120$. Solve the system for two iterations. **12**

9. Consider the main control volume shown in figure 3. A staggered mesh is used with the u and v velocity components stored as shown. The following quantities are given.
$u_W = 7$, $v_S = 3$, $p_N = 0$ and $p_E = 50$. The flow is steady and the density is constant. The momentum equations for u_e and v_n are given by:
$u_e = D_e(p_E - p_C)$; $v_n = D_n(p_N - p_C)$
Also given $D_e = 2$, $D_n = 1.6$, and the control volume has Δx = Δy = 1. Starting with a guessed value of $p_C = 30$, use the SIMPLE algorithm to find u_e and v_n. Do you feel any iteration will be required for the problem? Justify your answer.

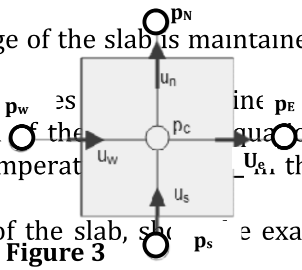

Figure 3

10 + 2 = 12