B.TECH/CHE/6TH SEM/CHEN 3204/2018
MATHEMATICAL METHODS IN CHEMICAL ENGINEERING
(CHEN 3204)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) One of the eigen vetors for the matrix $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$ is equal to

(a) $\begin{bmatrix}-0.7071\\ 0.7071\end{bmatrix}$ (b) $\begin{bmatrix}0.4472\\ -0.8944\end{bmatrix}$ (c) $\begin{bmatrix}0.3472\\ -0.6944\end{bmatrix}$ (d) either (a) or (b).

(ii) The rank of the matrix $A=\begin{bmatrix}-1 & 4 & 3 & 2\\ 3 & -3 & -4 & 1\\ 0 & 9 & 5 & 7\end{bmatrix}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3.

(iii) The radius of convergence for the series $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+........$ is equal to

(a) 1 (b) 2 (c) ∞ (d) 0.5.

(iv) In a heat conduction problem the temperature varies with position and time according to T = T(x,y,t). The correct representation of the problem is

(a) $\frac{\partial T}{\partial t} = \alpha\frac{\partial 2T}{\partial x2}$ (b) $\frac{\partial T}{\partial t} = \alpha\frac{\partial 2T}{\partial y2}$

(c) $\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial 2T}{\partial x2}+\frac{\partial 2T}{\partial y2}\right)$ (d) $\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial 2T}{\partial x2}+\frac{\partial 2T}{\partial y2}+\frac{\partial 2T}{\partial z2}\right)$.

(v) The skin friction drag coefficient inside a boundary layer on a flat plate is given by

(a) $C_d \propto \sqrt{Re}$ (b) $C_d \propto \frac{1}{\sqrt{Re}}$

(c) $C_d \propto Re$ (d) $C_d \propto \frac{1}{Re}$.

(vi) $(AB)^{-1}$ can be written as

(a) $A^{-1}B^{-1}$ (b) $B^{-1}A^{-1}$ (c) BA^{-1} (d) $A^{-1}B$.

(vii) The biorthogonal relation between two distinct eigen vectors y and x can be given by

(a) $y_i^{-1}x_j \begin{matrix}=0\,(\text{if } i=j)\\ \neq 0\,(\text{if } i\neq j)\end{matrix}\Big\}$ (b) $y_i^{-1}x_j \begin{matrix}=0\,(\text{if } i\neq j)\\ \neq 0\,(\text{if } i=j)\end{matrix}\Big\}$

(c) $y_i^{T}x_j \begin{matrix}=0\,(\text{if } i=j)\\ \neq 0\,(\text{if } i\neq j)\end{matrix}\Big\}$ (d) $y_i^{T}x_j \begin{matrix}=0\,(\text{if } i\neq j)\\ \neq 0\,(\text{if } i=j)\end{matrix}\Big\}$

(viii) Maclaurin series is actually synonymous with __________ series, when the function is expanded around x =0

(a)Fourier (b) Fourier-Bessel

(c)Taylor (d) Legendre.

(ix) **δ/l** for a boundary layer is proportional to

(a) 1/Re (b) Re (c) 1/√Re (d) $Re^{3/2}$.

(x) For large values of Reynolds number the disturbance thickness (**δ**) of a boundary layer approaches

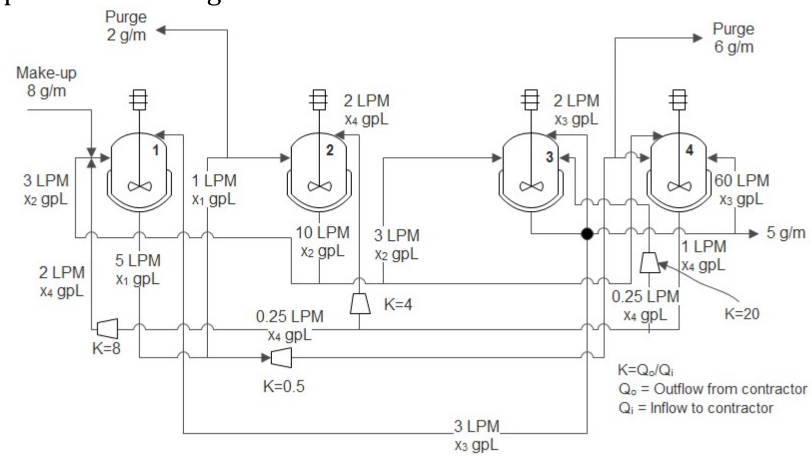
(a) 0 (b) ∞ (c) the length of the plate (l)

(d) 0.664 l.

Group – B

2. (a) Show that the eigen vector for a matrix 'A' is same as adj(A-λI), where λ is the eigen value.

(b) Using Gauss-Jordon elimination find out the outlet concentration (x_i, i=1,2,3,4) from each of the mixing tank as shown in the below process flow diagram.



3 + 9= 12

3.(a) Using Sylvester's Theorem expand exp(At),

where $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

(b) In a two stage counter-current solvent extraction process the feed is introduced counter currently with the pure solvent. The feed and the solvent flow rate are 10 LPM and 5 LPM respectively. The concentration of the component to be extracted in the feed is 0.5 gpL. Assume the operation is under steady state condition.

(i) What will be the degrees of freedom for the above system to be evaluated?

(ii) Find out the concentration of the component present in both raffinate and solvent phase from each stage after extraction. Given: stage 1: partition coefficient = 0.2 and stage 2: partition coefficient = 0.4

6 + (1 + 5) = 12

Group – C

4. (a) Show that $J_{-n}(x)=(-1)^n J_n(x)$, when n is a positive integer.

(b) A constant temperature vessel is supported on a truncated cone which in turn is resting on another surface maintained at constant temperature. The small end of the support is of radius 4 cm and is at 50°C, the temperature of the vessel. The wide end of the support is of radius 5 cm and is at 10°C, the temperature of the surface. The vertical height of the support is 10 cm and its thermal conductivity is 70 W/mK. The curved surface of the support loses heat at a temperature 10°C, when the convective heat transfer coefficient is 10W/m²K. Assuming uniform radial temperature distribution within the support and by measuring all temperatures relative to the temperature of the base, calculate the first five terms in the series giving the temperature distribution within the support.

4 + 8= 12

5. (a) Determine the steady-state concentration distribution C(r) and the effectiveness factor (η) for an isothermal first-order reaction occurring in an infinitely long cylindrical porous catalyst pellet of radius R. There is a finite external mass transfer resistance at the pellet surface for which at r=R

$D\frac{dC}{dr} = k_g\left(C_f - C\right)$, where C_f is reactant concentration in the bulk fluid, and k_g is the mass transfer coefficient.

The relevant ODE is $\frac{D}{r}\frac{d}{dr}\left(r\frac{dC}{dr}\right) = kC, r \in \{0,R\}$, where D is the diffusion coefficient of reactant in the pellet, and k is the reaction rate constant.

Given: $\eta = \dfrac{\int_0^R rkC(r)\,dr}{\int_0^R rkC_f\,dr}$

(b) Find out the radius of convergence for a series $1+x+x^2+x^3+\ldots\ldots$ using both root and ratio method.

10 + 2 = 12

Group – D

6. (a) Frame an initial value problem involving heat conduction with appropriate conditions.

(b) Give an example of the mixed type of boundary condition applied in a heat transfer problem.

6 + 6 = 12

7. (a) Frame a boundary value problem involving unsteady state heat conduction. State the appropriate conditions also.

(b) Prove that sin nx and sin mx are orthogonal with respect to unity in the range $0 \le x \le \pi$ for integer values of m and n.

6 + 6 = 12

Group – D

8. Derive the Prandtl boundary layer equations for flow past a cylindrical body of length **l**. Assume that the boundary layer thickness is much much smaller than **l.**

12

9. What is the significance of Damkohler number in connection to diffusion with chemical reaction? Explain how boundary layer concept can be applied to describe the situation when diffusion takes place along with chemical reaction for an isothermal laminar flow along a flat plate.

3 + 9 = 12