MATHEMATICS II
(MATH 1201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The order, degree and linearity of the differential equation $x^3 \frac{d^2 y}{dx^2} + \cos x \frac{dy}{dx} + (\sin x) y = 0$ are respectively
(a) 2, 1, linear (b) 2, 1, non-linear
(c) 1, 2, linear (d) 1, 2, non-linear.

(ii) Integrating factor of $\frac{dy}{dx} + y \cot x = 2x \cos x$ is
(a) $\cos x$ (b) $\sin x$
(c) $-\sin x$ (d) none of these.

(iii) Let A=(1,3,5), B=(6,4,3), C=(-2,-1,4) and D=(0,1,5). The projection of AB on CD is
(a) $-\frac{10}{3}$ (b) $\frac{10}{3}$ (c) $\frac{10}{6}$ (d) $\frac{5}{3}$

(iv) The equation of the plane passing through (1, 2, 3) and parallel to $2x + 3y - z + 5 = 0$ is
(a) $2x + 3y - z + 7 = 0$ (b) $2x + 3y - z + 5 = 0$
(c) $2x + 3y - z - 5 = 0$ (d) $x + y + z + 5 = 0$

(v) A vertex of a tree is a cut vertex if its degree is
(a) greater than 1 (b) greater than 2
(c) equal to 1 (d) equal to 2.

(vi) In a connected simple graph with 8 vertices and e edges
(a) $7 \le e \le 26$ (b) $6 \le e \le 28$
(c) $7 \le e \le \infty$ (d) $7 \le e \le 28$

(vii) If $L\left\{\frac{\sin t}{t}\right\} = tan^{-1}\left(1/s\right)$, then $L\left\{\frac{\sin at}{t}\right\} =$
(a) $tan^{-1}\left(1/s^2\right)$ (b) $tan^{-1}\left(a/s\right)$
(c) $tan^{-1}\left(1/as\right)$ (d) $tan^{-1}\left(\frac{1}{s^2+a^2}\right)$

(viii) $L\{e^{-2t}\cos t\}$ is
(a) $\frac{s+2}{s^2+4s+5}$ (b) $\frac{s}{s^2+4s+5}$
(c) $\frac{s+1}{s^2+4s+1}$ (d) $\frac{s+3}{s^2+4s+5}$

(ix) The value of $\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{5}{2}\right)$ is
(a) $\frac{3\sqrt{\pi}}{4}$ (b) $\frac{3\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

(x) A line on the xy-plane makes an angle 30° with x-axis. Then the direction cosine of this line are
(a) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$ (b) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ (c) $\left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$ (d) $\left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Group – B

2. (a) Find the integrating factor of the following equation: $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

(b) Find general and singular solution of the following differential equation: $p = \cos(y - px)$, $p \equiv \frac{dy}{dx}$

(c) Solve $\frac{dy}{dx} = y \tan x - y^2 \sec x$.

2 + 5 + 5 = 12

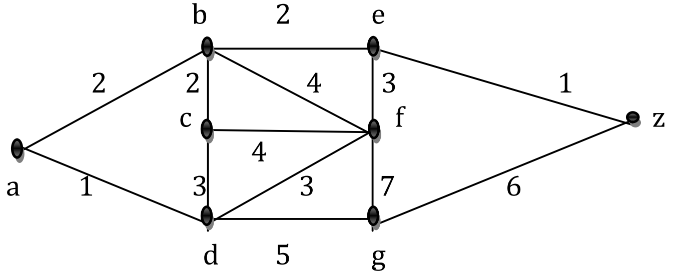
3. (a) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 3\sin x + 4\cos x$, $y(0) = 1$, and $y'(0) = 0$ using D operator method.

(b) Solve by the method of variation of parameters. $(D^2 - 2D + 1)y = e^x \log x$, where $D \equiv \frac{d}{dx}$.

6 + 6 = 12

Group – C

4. (a) Find the shortest path between a to z by Dijkstra's algorithm.



(b) Draw the graph whose incidence matrix is $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$.

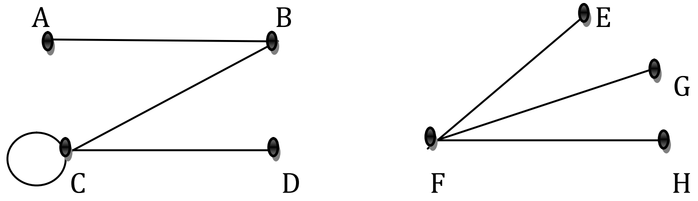
(c) Does the graph with the given degree sequence exist? Justify your answer. $\{6, 6, 6, 4, 3, 2\}$.

6 + 4 + 2 = 12

5. (a) Prove that any tree with two or more vertices contains at least two pendant vertices.

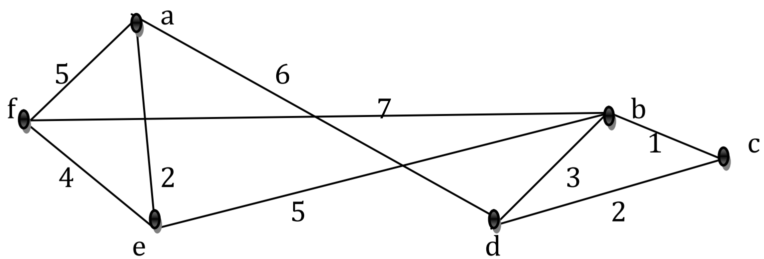
(b) Construct the adjacency matrix of the following disconnected graph, having two components I_1 and I_2,

I:



I_1 I_2

(c) Find the minimal spanning tree of the following graph by Prim's algorithm.



5 + 2 + 5 = 12

Group – D

6. (a) Show that $\int_0^\infty \frac{dx}{(x+1)(x+2)} = \log 2$.

(b) Assuming $\Gamma(m)\Gamma(1-m) = \pi \operatorname{cosec}(m\pi)$, $0 < m < 1$, show that $\Gamma\left(\frac{1}{9}\right).\Gamma\left(\frac{2}{9}\right).....\Gamma\left(\frac{8}{9}\right) = \frac{16}{3}\pi^4$.

(c) Evaluate: $\int_{-1}^{1} \frac{\sqrt{1+x}}{\sqrt{1-x}}$ dx, if it exists.

3 + 4 + 5 = 12

7. (a) Evaluate $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\}$.

(b) Solve the following differential equation, using Laplace transform method: $y''(t) + y(t) = 8\cos t$, where $y(0) = 1, y'(0) = -1$.

6 + 6 = 12

Group – E

8. (a) If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

(b) Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $x + 4y - 6z + 1 = 0$. Find the equation of the straight line joining the origin and the foot of the perpendicular.

6 + 6 = 12

9. (a) Find the equation of the projection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$ on the plane $x + 3y + z + 5 = 0$.

(b) Find the equation of the plane which contains the line of intersection of the planes $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and perpendicular to the xy-plane.

6 + 6 = 12