B.TECH/ CSE /4TH SEM/ MATH 2202/2018

PROBABILITY AND NUMERICAL METHODS

(MATH 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $P(A) = 0.5$, $P(A \cap B^c) = 0.4$, then $P(A \cap B) =$
(a) 0.3 (b) 0.9 (c) 0.1 (d) 0.7.

(ii) If the probability of a screw being defective is $p = 0.015$, then the probability that a box of 100 screws does not contain a defective one is
(a) $1-(0.015)^{100}$ (b) $(0.985)^{100}$ (c) $(0.015)^{100}$ (d) $1-(0.985)^{100}$.

(iii) $(\Delta - \nabla)x^2$ is equal to
(a) h^2 (b) $-2h^2$ (c) $2h^2$ (d) h^{20}.

(iv) We wish to solve $x^2 - 2 = 0$ by Newton-Raphson technique. If initial guess is $x_0 = 1$, then the immediate estimate of x (*i.e.*, x_1) will be
(a) 1.414 (b) 1.5 (c) 0 (d) -1.

(v) Which of the following matrices is a probability transition matrix ?
(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{3}{4} & 0 & \frac{1}{4} \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & -1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

(vi) If $\frac{dy}{dx} = x + y$ and $y(1) = 0$, then $y(1.1)$ by Euler's method is $[h = 0.1]$:
(a) 0.1 (b) 0.3 (c) 0.5 (d) 0.9.

(vii) Condition for convergence of Newton- Raphson method is
(a) $|f(x)\,f'(x)| < \{f''(x)\}^2$ (b) $|f(x)\,f''(x)| < \{f'(x)\}^2$
(c) $|f(x)\,f'(x)| > \{f''(x)\}^2$ (d) $|f(x)\,f''(x)| > \{f'(x)\}^2$

(viii) The probability of obtaining the sum as 10, when two dice are thrown is
(a) 3/36 (b)2/36
(c) 1/36 (d) 5/36.

(ix) The function $f(x,y) = \begin{cases} kxy(x+y), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$ can be a p.d.f. if the value of is
(a) 3 (b) -3 (c)1/3 (d) 2.

(x) The degree of precision of Simpson's one-third rule is
(a) 1 (b) 2 (c) 3 (d) 5.

Group – B

2. (a) Find the root of the equation $x^3 - 5x - 7 = 0$ that lies between 2 and 3 using Regula-Falsi method correct up to 2 significant figures.

(b) Does the following system of equations satisfy the sufficient conditions for the convergence of Gauss-Seidel iterative method?

$$-x_2 - x_3 + 6x_1 = 11.33$$
$$-x_1 - x_3 + 6x_2 = 32$$
$$-x_2 - x_1 + 6x_3 = 42$$

If it satisfies, then use the method to solve the system of equations (Perform 3 iterative steps).

6 + 6 =12

3. (a) Compute $f(0.23)$, from the following table, by using Newton Forward interpolation formula, correct up to three decimal places.

x	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

(b) Evaluate $\int_0^{0.6} x \sin x^2 \, dx$, using Trapezoidal method by taking $h = 0.1$ correct up to five decimal places.

7 + 5 = 12

Group – C

4. (a) A number is randomly chosen from the set { 1 , 2 , 3, ...,99, 100 }. Find the probability that the number is divisible by 2 or 3. What is the probability that a number is divisible by 2 given that it is divisible by 3?

(b) Two unbiased dice are thrown. Find the conditional probability that two fives occur if it is known that the total sum is divisible by 5?

(c) Prove that if A and B are independent events then A^c and B^c are also independent.

6 + 3 + 3 =12

5. (a) When a computer goes down, there is a 75% chance that it is due to an overload and a 15% chance that it is due to a software problem. There is an 85% chance that it is due to an overload or software problem. What is the probability that both of these problems are at fault? What is the probability that there is a software problem but no overload?

(b) Assume that 95% of all cryptographic messages are authentic. Furthermore, assume that only 0.1% of all unauthentic messages are sent using the correct key and that all authentic messages are sent using the correct key. Find the probability that a message is authentic given that the correct key is used.

(c) Two dice are thrown. What is the probability that the product of the numbers appearing in the two faces is an even number?

4 + 5 + 3 = 12

Group – D

6. (a) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and standard deviation Rs. 70 and Rs. 5 respectively , find the number of workers whose weekly wages are
   (i) between Rs. 66 and Rs. 72
   (ii) less than Rs. 66
   (iii) more than Rs. 72

(b) Suppose a biased coin has probability of 0.7 of getting a head. The coin is tossed 3 times. Let X = Number of heads that appear in 3 tosses. Find the possible values that X can take and find its probability mass function. Find $E(X)$.

6 + 6 = 12

7. (a) A fair coin is tossed 5 times. Find the probability that there are
   (i) exactly 2 tails
   (ii) at most 2 tails
   (iii) at least 2 tails
   (iv) fewer than 2 tails

(b) If a ticket office can serve at most 4 customers per minute and the average number of customers is 120 per hour, what is the probability that during a given minute customers will have to wait?

(c) For two random variables X and Y with the same mean, the two regression lines are $y = ax + b$ and $x = \alpha y + \beta$.

Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$

4+ 4 + 4 = 12

Group – E

8. (a) A fair coin is tossed three times. Let X denote the number of heads in three tossings and Y denote the absolute difference between the number of heads and the number of tails. Find the joint p.m.f. of (X,Y). Also find the marginal p.m.f. of X and Y. Are these two random variables X and Y independent?

(b) The joint distribution of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{2}, & \text{if } (x,y) \in \mathcal{R} \\ 0, & \text{otherwise} \end{cases}$$

Where $\mathcal{R}$ is the interior of the triangle of area 2 square units having vertices $(0,0), (2,0)$ and $(1,2)$. Find the marginal density function of X. Also find $P(X \leq 1, Y \leq 1)$.

7 + 5 = 12

9. (a) A computer system can operate in two different modes, Mode-I and Mode-II. Every hour it remains in the same mode or switches to a different mode according to the transition probability matrix

$$P = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$$

(i) Compute the 2-step transition probability matrix.

(ii) If the system is in Mode-I at 5:30 pm, what is the probability that it will be in Mode-I at 8:30 pm on the same day?

(b) A communication system transmits the two digits 0 and 1, each of them passing through several stages. Suppose that the probability, that the digit that enters remains unchanged when it leaves, is p and that it changes is $q = 1 - p$. Suppose further that X_0 is the digit which enters the first stage of the system and $X_n, n \geq 1$ is the digit 0 leaving the n^{th} stage of the system. Then calculate and $P(X_2 = 0|X_0 = 1)$ and $P(X_3 = 1|X_0 = 0)$.

5 + 7 = 12