(v) Topological Sort is not allowed on a graph
(a) if the graph is a tree
(b) if the graph has a directed cycle
(c) if the underlying undirected graph has a cycle
(d) None of the above.

(vi) Which of the following is not O(n^2)?
(a) (15^10) * n + 12099 (b) n^1.98
(c) n^3 / (sqrt(n)) (d) (2^20) * n.

(vii) Let *S* be a sorted array of n integers. Let t(n) denote the best case time taken by an algorithm to determine if there are two elements with sum less than 1000 in *S*. which of the following statements is true?
(a) t(n) is O(1) (b) $n < t(n) < n\log_2 n$
(c) $n\log_2 n < t(n) < \binom{n}{2}$ (d) $t(n) = \binom{n}{2}$.

(viii) The characters a to h have the set of frequencies based on the first 8 Fibonacci numbers as follows:
a : 1, b : 1, c : 2, d : 3, e : 5, f : 8, g : 13, h : 21
A Huffman code is used to represent the characters. What is the sequence of characters corresponding to the code: 110111100111010?
(a) fdheg (b) ecgdf (c) dchfg (d) fehdg.

(ix) Consider the following recurrence equation:

$$T(n) = \begin{cases} k & \text{for } n = 1 \\ 3T(n/2)+kn & \text{for } n>1 \end{cases}$$

Which of the following statements is TRUE?
(a) $T(n) = O(n^{1.59})$ (b) O(1) (c) O(nlog n) (d) None of these.

(x) Running time of BUILD-MAX-HEAP function of Heap Sort is tightly bounded by which of the following?
(a) O(n) (b) O(n log n) (c) O(1) (d) None of these.

## Group – B

2. (a) Prove that a MIN-HEAP can be built in O(n) time from a given unordered array of n integers. You need not give the pseudo-code for building MIN-HEAP. Just give the time-complexity analysis.

(b) You are provided as input a positive integer *n*> 1 and an unsorted sequence S of *n* integers.





Hint: For operation i = 1, cost is 1,
operation i = 2, cost is 4,
operation i = 3, cost is 1,
operation i = 4, cost is 16,
operation i = 5, 6 ,7, cost is 1,
operation i = 8, cost is 64, and so on.

(b) Draw a state-transition diagram for string-matching automaton for the following pattern P over alphabet Σ = {a,b}. Be careful about careless mistakes as this is a simple question.
P = abbaabbabbaabb.

**7 + 5 = 12**

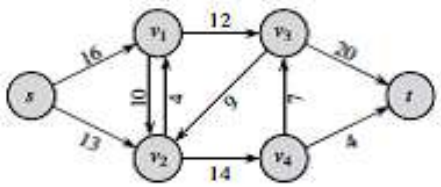
## Group – E

8. (a) Show that the Clique Decision Problem is NP-hard by using the result that the 3-CNF-SAT problem is computationally hard.

(b) What do you mean by approximation ratio of an approximation algorithm? In that context define polynomial-time approximation scheme and fully polynomial-time approximation scheme.

**6 + (2 + 2+ 2) = 12**

9. (a) What do you mean by augmenting path of a residual network?
Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represents capacity of the respective edge.




(b) State whether the following problems are NP-Hard or polynomial-time solvable :-
Set Cover Problem, Edge Cover Problem, Vertex Cover Problem, Eulerian Path Problem, Hamiltonian Path Problem, Finding the longest path.

**(2 + 7) +( 6 × 0.5) = 12**

# DESIGN & ANALYSIS OF ALGORITHMS
## (CSEN 2201)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
## (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Time complexity for finding the longest common subsequence of two sequences of length m and n using dynamic programming method is
(a) O(mlogm + nlogn) (b) O(mn)
(c) O(mnlogm) (d) O(mlogn + nlogm).

(ii) A student proved that the longest path problem is NP-complete by reducing it to another already known NP-complete problem named Graph Isomorphism problem. The teacher did not give him any marks. What could be the reason?
(a) The teacher committed a mistake.
(b) The student got a wrong answer about the hardness of longest path problem.
(c) Graph Isomorphism is actually solvable in polynomial time.
(d) None of the above is true.

(iii) For which of the following problem an approximation scheme exists?
(a) Vertex Cover problem
(b) Finding a Hamiltonian circuit in a given graph
(c) 0-1 Knapsack problem
(d) None of the above.

(iv) The time complexity of Counting sort with number of elements **n** and maximum element **m** is
(a) O(m + log n) (b) O(n + log m)
(c) O(log m log n) (d) None of the above.

i) Show that you can find both the 1st and nth order statistic simultaneously in exactly $\lceil 3n/2 \rceil - 2$ number of pair-wise comparisons.
Hint: This is not at all difficult. Just tackle the odd and even case separately.

ii) Further show that you can find *both* the second order statistic and the $n^{th}$ order statistic of S, in $(\lceil 3n/2 \rceil + \lceil \mathbf{lgn} \rceil - 3)$ number of pair-wise comparisons.

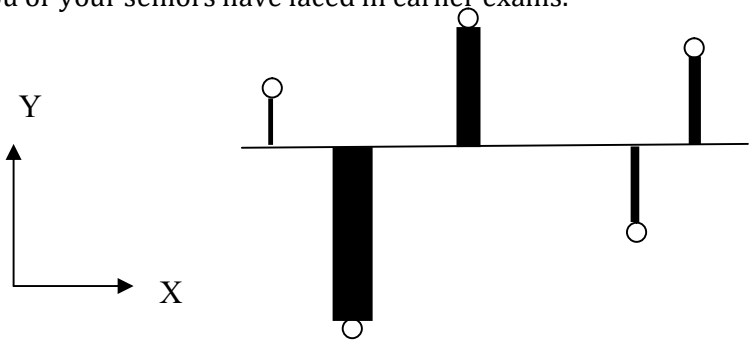
**5 +(4 + 3) = 12**

3. (a) Mr. Nano is a VLSI engineer. He wants to connect **n** circuit points to the clock signal. The clock signal passes through a wire that is parallel to the x-axis. The circuit points are to be connected by wires that are all vertical to the clock line. Suppose the design rule specifies that the **width ($w_i$) of each vertical wire cannot be made less than quarter of its length**. Please look at the adjoining figure to have a feel. Note that, the width of the lines are not drawn to the scale, but it still reflects the main issue, the longer wires need to be wider also.

Now, if the coordinates ($x_i$, $y_i$ ) for each circuit point $c_i$ to be connected are given, find the place through which the clock line should pass, i.e. find the equation of the clock line $y = y_c$ , so as to minimize the total material **L**(may be gold for excellent signal speed) used to route these vertical wires, where $\mathbf{L} = \sum_{i=1}^{n} w_i(y_i - y_c)$.

Also give a simple proof that the answer you found is optimal.

**Hint:** Note carefully that this question **is not** exactly the same as what you or your seniors have faced in earlier exams.




(b) Suppose we have a O(n) time deterministic algorithm that finds median of an unsorted array. Now consider a QuickSort implementation where at every stage we first find the median using the above algorithm, then use that median as a pivot. What will be the worst case time complexity of this modified QuickSort? Your answer should be tight. Also justify your answer.

**(2 + 5) + (1 + 4)= 12**





## Group – C

4.(a) (i) Name an algorithm that solves the all-pair-shortest-path problem using dynamic programming.

(ii) Give the pseudo-code for that algorithm and also state its complexity.

(iii) Why should we use this algorithm instead of running the single-source-shortest-path problem n times to solve the same?

(b) Gunodhar is multiplying 100 square matrices each having 40000 elements. He has a choice of using either the matrix-chain-order multiplication method that uses an $O(n^3)$ matrix multiplication algorithm within itself or just a faster matrix multiplication algorithm named Strassen's algorithm of complexity $O(n^{2.8})$. Which choice will be better for him if he wants to save time in getting the job done?

**(1+7+2) +2= 12**

5 (a) Consider an undirected graph G (V, E) with the set of vertices $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. E is a set of edges of the form (u, v, w) which denotes an undirected edge between u & v with weight w. $E = \{(v_1, v_2, 4), (v_2, v_3, 2), (v_2, v_4, 6), (v_3, v_1, 3), (v_3, v_4, 2), (v_1, v_5, 4), (v_4, v_5, 3), (v_5, v_6, 1), (v_5, v_7, 3), (v_6, v_3, 4), (v_2, v_7, 5), (v_4, v_7, 3)\}$. Illustrate the steps of finding a minimum spanning tree of the given graph using Kruskal's algorithm implemented by disjoint set data structure.

(b) Prove that Kruskal's algorithm always returns a minimum spanning tree.

**7 + 5 =12**

## Group – D

6.(a) Define single source shortest path problem. Prove that shortest path problem follows optimal sub-structure property.

(b) Give the pseudo code for dijkstra's algorithm, implemented by priority queue. In which situation you can't apply dijkstra's algorithm?

(c) Define strongly connected component of a given graph.

**(1 + 2) +(6 + 1) + 2=12**

7. (a) A sequence of *n* operations is performed on a data structure. The *i*th operation costs

| | |
|---|---|
| $i^2$ | if *i* is a power of 2, |
| 1 | otherwise. |

Use aggregate analysis to determine the value of k such that the amortized cost per operation lies between k and k + 1.

i) Show that you can find both the 1st and nth order statistic simultaneously in exactly $\lceil 3n/2 \rceil - 2$ number of pair-wise comparisons.
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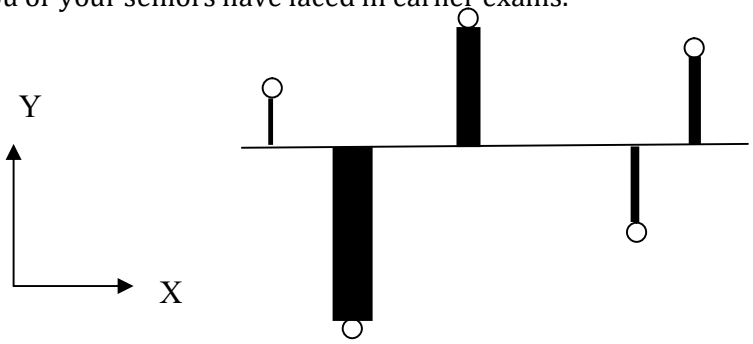


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