B.TECH/AEIE/CSE/8TH SEM/MATH 4282/2018

ADVANCED COMPUTATIONAL MATHEMATICS AND GRAPH THEORY (MATH 4282)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) $(1! + 2! + 3! + 4! + \cdots + 99! + 100!)\ mod 10 =$
(a) 4 (b) 3 (c) 2 (d) 1

(ii) $30!\ mod\ 31 =$
(a) 31 (b) 1 (c) 29 (d) 30

(iii) $\varphi(123456789) =$
(a) 123 (b) 2345 (c) 79467 (d) none of these

(iv) $\left\{ {4 \atop 2} \right\} =$
(a) 6 (b) 8 (c) 7 (d) none of these

(v) For any integer $n > 0$, $\left[{n \atop 1} \right] =$
(a) $n!$ (b) $(n+1)!$ (c) $(n-2)!$ (d) $(n-1)!$

(vi) The fourth Bernoulli number $B_4 =$
(a) 0 (b) $-\frac{1}{30}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

(vii) $\left\langle {4 \atop 2} \right\rangle =$
(a) 8 (b) 10 (c) 11 (d) 12

(viii) Which one of the following is not a Fibonacci number?
(a) 55 (b) 233 (c) 144 (d) 376

(ix) The chromatic number of a circuit having 37 vertices is
(a) 36 (b) 37 (c) 2 (d) 3

(x) A connected planar graph has 5 edges and 3 vertices. The number of faces in the graph is
(a) 4 (b) 2 (c) 5 (d) 1.

Group – B

2. (a) Show that
$$\sum_{1\le j<k\le n} \frac{1}{k-j} = \sum_{0\le l<n} H_l$$

(b) Evaluate the sum $\sum_{k=1}^{n} \frac{(-1)^k k}{(4k^2-1)}$.

6 + 6 = 12

3. (a) Solve the following recurrence: $Q_0 = \alpha$, $Q_1 = \beta$; $Q_n = (1 + Q_{n-1}) / Q_{n-2}$ for $n > 1$. Assume that $Q_n \neq 0$ for all $n \geq 0$. Find Q_2, Q_3, Q_4, Q_5, Q_6, Q_{1003} and Q_{1004}. Show your calculations in detail and justify your answer.

(b) Evaluate the sums $S_n = \sum_{k=0}^{n}(-1)^{n-k}$ and $T_n = \sum_{k=0}^{n}(-1)^{n-k}k$ assuming that $n \geq 0$.

6 + 6 = 12

Group – C

4. (a) Write down the sequence of the first twenty Fibonacci numbers and discover a relation between gcd (F_m, F_n) and $F_{\gcd(m,n)}$. Verify it for any eight (m, n) pairs. Show your work in detail.

(b) Prove that $\sum_{k=0}^{n} \binom{r+k}{k} = \binom{r+n+1}{n}$ where n and r are positive integers.

6 + 6 = 12

5. (a) State the definition of Stirling numbers of the first kind, denoted by $\left[{n \atop k} \right]$. Prove that $\sum_{k=0}^{n} \left[{n \atop k} \right] = n!$

(b) State the definition of Eulerian numbers, denoted by $\left\langle {n \atop k} \right\rangle$. Find a recurrence relation for the $\left\langle {n \atop k} \right\rangle$. Prove it.

6 + 6 = 12

Group – D

6. (a) Prove that $n^{n/2} \leq n! \leq \frac{(n+1)^n}{2^n}$, where n is a positive integer.

(b) Find the greatest common divisor of 11698 and 92 by using the Euclidean algorithm. Express it as $11698x + 92y$ where x and y are integers. Show your work in detail.

6 + 6 = 12

7. (a) State and prove Fermat's Little Theorem.

(b) Find (3^{302}+4^{203}) mod 7. Show your calculations in detail.

6 + 6 = 12

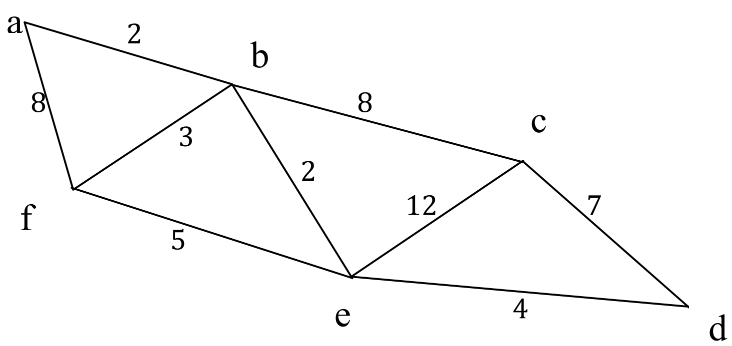
Group – E

8. (a) If a graph is the union of K_4 and a triangle in such a way that they share only a common vertex, then what will be the chromatic polynomial of the graph?

(b) If G is a connected planar graph having 20 faces and the degree of every vertex of it is 3, find the number of vertices of G.

6 + 6 = 12

9. (a) Apply the Kruskal's algorithm to find the spanning tree of the following graph. Find the weight of the spanning tree.



(b) Define matching and perfect matching. Write down all the perfect matchings in K_6 (complete graph having six vertices).

6 + 6 = 12