FUNDAMENTALS OF DIGITAL SIGNAL PROCESSING
(AEIE 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For energy signals, the average power will be
(a) infinite (b) finite
(c) zero (d) cannot be defined.

(ii) An LTI discrete time system is causal, if and only if,
(a) $h(n) \neq 0$ for $n<0$ (b) $h(n) = 0$ for $n<0$
(c) $h(n) \neq \infty$ for $n<0$ (d) $h(n) \neq 0$ for $n>0$.

(iii) The fundamental period of the sequence $x(n) = \cos\left(\frac{2\pi}{3}\right)n$ is
(a) 1 (b) 2 (c) 3 (d) 6.

(iv) In a signal $x(n)$, if 'n' is replaced by $\frac{n}{3}$, then it is called,
(a) upsampling (b) folded version
(c) downsampling (d) shifted version.

(v) If Z-transform of $x(n)$ is $X(z)$, then Z-transform of $x(-n)$ is
(a) $-X(z)$ (b) $X(-z)$
(c) $-X(z^{-1})$ (d) $X(z^{-1})$

(vi) The second order LTI discrete time systems behave as,
(a) low pass filter (b) high pass filter
(c) resonant filter (d) all pass filter.

(vii) The Z-transform of $\delta(n)$ is
(a) 0 (b) Z^{-1} (c) $\frac{1}{1-Z^{-1}}$ (d) 1.

(viii) In an N-point DFT of a finite duration sequence $x(n)$ of length L, the value of N should be such that _____,
(a) $N \geq L$ (b) $N \neq L$
(c) $N \leq L$ (d) $N = L$

(ix) The condition for the impulse response to be antisymmetric is,
(a) $h(n) = -h(N-1-n)$ (b) $h(n) = h(-n)$
(c) $h(n) = h(N-1-n)$ (d) $h(n) = h\left(\frac{N-1}{2}\right)$

(x) The zeros of the Butterworth filters exist at
(a) left half of s-plane (b) Origin
(c) Infinity (d) Right half of s-plane.

Group – B

2. (a) Determine whether the following system is time-invariant or time variant: $y(n) = x(-n)$

(b) What are the properties of linear convolution?

(c) Find the circular convolution of the two sequences $x_1(n) = \{1, 2, 2, 1\}$ and $x_2(n) = \{1, 2, 3, 1\}$.

3 + 3 + 6 = 12

3. (a) Determine whether the following signal is energy or power signal. $x(n) = \sin\left(\frac{\pi}{3}n\right)$

(b) Test the stability of LTI system, whose impulse response is $h(n) = 0.2^n u(-n) + 3^n u(-n)$.

(c) Let an analog signal, be $x_a(t) = 6\cos 50\pi t + 3\sin 200\pi t - 3\cos 100\pi t$. Determine the minimum sampling frequency and the sampled version of analog signal at this frequency.

4 + 4 + 4 = 12

Group – C

4. (a) Find the Z-transform of $x(n) = \cos(n\theta)u(n)$.

(b) Using residue method find the inverse Z–transform of $X(z) = \dfrac{z}{(z-1)(z-2)}$ where $1 < |z| < 2$.

(c) What are the properties of Region of convergence?

5 + 5 + 2 = 12

5. (a) Find the system transfer function *H (z)* and the impulse response *h (n)* of the system described by the difference equation $y(n) = x(n) + 2x(n-1) - 4x(n-2) + x(n-3)$.

(b) Prove, if $X_+(z) = Z\{x(n)\}$, then $Z\{x(n+m)\} = z^m \left\{ X_+(z) - \sum_{k=0}^{m-1} x(k) z^{-k} \right\}$

where *m* is a positive integer.

(c) State the initial value theorem.

5 + 5 + 2 = 12

Group – D

6. (a) Find the DFT of a sequence $x(n) = \{-1,2,2,2,-1\}$ using 8 point DIT FFT algorithm.

(b) How many multiplications and additions are involved in N-point radix-2 FFT?

10 + 2 = 12

7. (a) Compute linear convolution of the following two sequences using DFT: $x(n) = \{\underset{\uparrow}{1},2\}$ and $h(n) = \{\underset{\uparrow}{2},1\}$.

(b) Prove, if $DFT\{x(n)\} = X(k)$ then $DFT\{x(N-n)\} = X(N-k)$.

8 + 4 = 12

Group – E

8. (a) Determine the order and the poles of low pass Butterworth filter that has a 3dB attenuation at 500 Hz and an attenuation of 40dB at 1000Hz.

(b) Obtain the direct form-I realization for the system described by difference equation $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$.

6 + 6 = 12

9. (a) For the analog transfer function $H(s) = \dfrac{2}{s^2 + 3s + 2}$ determine $H(z)$ using impulse invariance method. Assume *T*=1 sec.

(b) Realize the following system with minimum number of multipliers: $H(z) = \dfrac{1}{3} + \dfrac{1}{4}z^{-1} + \dfrac{3}{2}z^{-2} + \dfrac{3}{2}z^{-3} + \dfrac{1}{4}z^{-4} + \dfrac{1}{3}z^{-5}$.

6 + 6 = 12