

M.TECH/CSE/1st SEM/MATH 5102/2017
ADVANCED DISCRETE MATHEMATICS
(MATH 5102)

Time Allotted: 3 hrs

Full Marks: 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and

Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group - A

(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**
 - (i) For all natural number n , the number $n^2 + n$ is

(a) even	(b) odd
(c) prime	(d) irrational.
 - (ii) The greatest element in $\{1, 2, 3, 4, 6, 12, 18\}$ with respect to divisibility is

(a) 12	(b) 18
(c) 6	(d) there exists no greatest element.
 - (iii) The relation a divides b on the set of integers is

(a) equivalent relation	(b) partial order relation
(c) symmetric	(d) none of these.
 - (iv) Total number of non-negative integral solutions to the equation $x + y + z = 17, x, y, z \geq 0$ is

(a) 171	(b) 170
(c) 169	(d) None of these.
 - (v) If $17 \equiv 7 \pmod{p}$, then p can be

(a) 5	(b) 6	(c) 7	(d) 17.
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 - (vi) The function $\frac{2}{1-2x} + \frac{1}{1-x}$ generates the sequence $\{a_n\}$ where a_n is equal to

(a) $2^{n+1} + 1$	(b) $2^n + 1$	(c) $2^{n+1} - 1$	(d) $2^n - 1$.
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 - (vii) The sum of the co-efficient in the expansion of $(x + y + z)^{10}$ is

(a) 2^{10}	(b) 2^9	(c) 3^{10}	(d) 3^9 .
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- (viii) A tree has 4 vertices of degree 2, 3 vertices of degree 3, 3 vertices of degree 4. How many pendant vertices the tree should have?

(a) 11	(b) 10	(c) 9	(d) 12
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- (ix) Clique number for a complete graph K_n with n vertices is

(a) 0	(b) 1	(c) 2	(d) n
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- (x) The chromatic number of the Kuratowski's first graph is

(a) 3	(b) 4	(c) 5	(d) none of these.
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Group - B

2. (a) Let R be a binary relation on the set of all strings of 0's and 1's such that $R = \{(a, b) : a \& b \text{ are strings that have same number of 0's}\}$. Is R reflexive, symmetric, transitive or a partial order relation?
 - (b) Let $A = \{1, 2, 3, 5, 30\}$ and $a \rho b$ if and only if a divides b . Find the greatest and least element.
 - (c) Let D_m denote the positive divisors of m ordered by divisibility. Draw the Hasse diagram of (i) D_{15} (ii) D_{16} .

5 + 2 + 5 = 12
3. (a) Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Determine a relation R on A by $aRb \Leftrightarrow 3$ divides $(a - b)$. Show that R is an equivalence relation. Also determine the partitions generated by R .
 - (b) Let S be any non empty set and $P(S)$ be its power set. Show that $(P(S), \subseteq)$ is a lattice.

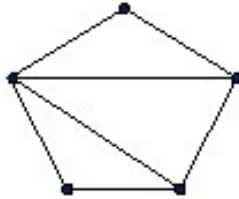
6 + 6 = 12

Group - C

4. (a) Use Mathematical induction to prove that $(3 + \sqrt{7})^n + (3 - \sqrt{7})^n$ is an even integer $\forall n \in \mathbb{N}$.
 - (b) If a is prime to b , then prove that a^2 is prime to b .
 - (c) Use division algorithm to prove that the square of an odd integer is of the form $8k + 1$, where k is an integer.

5 + 2 + 5 = 12

5. (a) Use Euclidean algorithm to find integers u and v such that $\gcd(63, 55) = 63u + 55v$.
8. (a) State and prove Euler's Theorem for planar Graph.
- (b) Using Decision problem, prove that the chromatic polynomial of the following graph is $b(b-1)(b-2)$, and $c \equiv d \pmod{m}$, then $(a +$
- (c) Prove that $19^{20} \equiv 1$

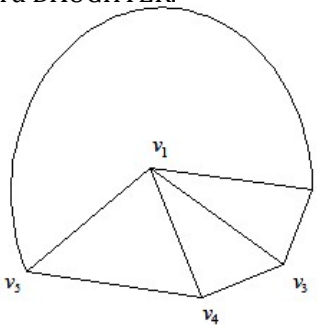


$$5 + 4 + 3 = 12$$

6. (a) A person writes 1 and addresses on the corresponding envelopes. In how many ways can the letters be placed in the envelopes so that
- (i) all the letters are in the wrong envelopes.
- (ii) at least two of them are in the wrong envelopes.
9. (a) Determine the chromatic polynomial of the graph
- (b) Find the number of permutations that can be had made from the letters of the word DAUGHTER.

$$6 + 6 = 12$$

- (i) taking all
(ii) beginning
(iii) beginning
(iv) vowels
(v) not all vowels



- (c) If there are 200 that speak Russian, 100 that speak French only, 10 that speak French or Russian, and 60 that speak Spanish, while 100 that speak either French or Russian, how many speak either French or Russian?
- (b) Prove that a graph (with at least one edge) is 2-chromatic if and only if it has no circuits of odd length.

$$4 + 5 + 3 = 12$$

7. (a) Solve the recurrence relation $a_n = 3a_{n-1} + 2a_{n-2}$, $a_0 = 1, a_1 = 1$.
- (b) Determine the number of integers between 1 and 10,000,000 (both inclusive) having the sum of digits equal to 18.

$$6 + 4 + 2 = 12$$

$$6 + 6 = 12$$