ALGORITHMS & COMPLEXITY
(CSEN 5102)

Time Allotted: 3 hrs **Full Marks: 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Consider the following code snippet which finds greatest common divisor (gcd) of two integers n & m, where n ≥ m:

```
int gcd(n, m)
 {
  if (n%m==0) return m;
n=n%m;
return gcd(m, n);
}
```

How many recursive calls are made by the above gcd function?
(a) θ(logn) (b) Ω (n) (c) θ(loglogn) (d) θ(sqrt(n)).

(ii) The Single-source shortest path problem in a graph G becomes undefined if there exists _____from the source.
(a) a –ve weight cycle in the graph
(b) a –ve edge reachable from the source
(c) multiple –ve edges in the graph
(d) a –ve weight cycle reachable from the source.

(iii) Assuming T(i) = 1, the solution of the recurrence relation is $T(n) = 2T(\lfloor n/2 \rfloor) + n$
(a) $O(n \log_2 n)$ (b) $\Omega(n \log_2 n)$
(c) $\Theta(n \log_2 n)$ (d) none of the above.

(iv) For simultaneously finding the minimum and maximum of n elements the number of comparisons required is at most
(a) $3\lfloor n/2 \rfloor$ (b) 2n (c) 2 (n -1) (d) n log n.

(v) Which of the given options provides the increasing order of asymptotic complexity of functions f_1, f_2, f_3 and f_4? (assume n ≥ 3)
$f_1(n) = 2^n$, $f_2(n) = n^{(3/2)}$, $f_3(n) = n\text{Logn}$, $f_4(n) = n^{(\text{Logn})}$
(a) f_3, f_2, f_4, f_1 (b) f_3, f_2, f_1, f_4 (c) f_2, f_3, f_1, f_4 (d) f_2, f_3, f_4, f_1.

(vi) In a flow network, there exists a cut of value 100. Which of the following can be concluded?
(a) The minimum flow in the network is 100.
(b) The maximum flow in the network ≤ 100.
(c) The maximum flow in the network ≥ 100.
(d) None of these.

(vii) Which of the following is not true about comparison based sorting algorithms?
(a) The minimum possible time complexity of a comparison based sorting algorithm is O(n log n) for a random input array
(b) Any comparison based sorting algorithm can be made stable by using position as a criteria when two elements are compared
(c) Counting Sort is not a comparison based sorting algorithm
(d) Heap Sort is not a comparison based sorting algorithm.

(viii) Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?
(a) R is NP-complete (b) R is NP-hard
(c) Q is NP-complete (d) Q is NP-hard.

(ix) Which of the following basic algorithms can be used to most efficiently determine the presence of a cycle in a given graph
(a) MST (b) Ford-Fulkerson (c) BFS (d)DFS.

(x) Topological Sort cannot be done in a directed graph of n vertices if the graph contains
(a) more than n edges (b) contains a directed cycle.
(c) contains a rooted tree (d) all of the above.

Group - B

2. (a) Do an average case analysis for Binary Search on a sorted array of distinct integers.

(b) Explain briefly (preferably with examples) that Amortized Analysis is not same as Average case analysis.

(c) You are given a sorted array of n distinct integers. When do the best-case and worst case happen in your Binary search and just state what is the number of comparisons required in each case?

7 + 3 + 2 = 12

3. (a) A sequence of n operations is performed on a data structure. The cost of ith operation is C(i) = i^2, if i is an exact power of 3
= 3, otherwise.
Calculate the exact expression for cost for n successive operations. Use Aggregate Analysis to determine the amortized cost per operation.

(b) Linear Search algorithm requires two comparisons per iteration. Modify the algorithm using sentinel element in such a way that the number of comparisons is reduced to one comparison at every iteration. Justify that this modified approach improves the performance of Linear Search by a constant factor.

6 +(4 + 2) = 12

Group - C

4. (a) Mention basic principles of any Divide-and-Conquer algorithm. Suppose you are given a set of n numbers. Write Quick Sort algorithm to sort those n numbers in ascending order, where last element is always chosen as pivot element. Now analyze the performance of your algorithm in best case & worst case.

(b) Heapsort and Mergesort have the same worst-case time complexities. Is there any reason why Heapsort may be considered better compared to Mergesort?

(2 + 4 + 4) + 2 = 12

5. (a) Define a Binary Search Tree (BST). When do you refer a Binary Search Tree as an AVL tree?

(b) Prove that an AVL tree with n nodes has height O (lg n).

(c) Assume that you have numbers between 1 and 1000 in a Binary Search Tree (BST). Now you want to delete a number from that BST. Write an algorithm to do this operation.

(2 + 2) +3 + 5 = 12

Group - D

6. (a) What do you mean by a spanning tree of a graph?
Suppose you are given a graph G = (V, E). Give the pseudo code for Kruskal's algorithm to find the minimal spanning tree of the graph G, where UNION-FIND data structure is used. Also explain the working principle of the algorithm in brief. Analyze the performance of the algorithm in the worst case assuming that the disjoint-set data structure is implemented by union-by-rank and path-compression heuristics.

(b) Suppose you are given a procedure DFS(G) that finds depth first search of a graph G. Explain how the procedure DFS(G) can be used to find the strongly connected components of a graph G.

(2 + 5 + 2) + 3 =12

7. (a) Explain the working principle of Bellman Ford algorithm for single source shortest path problem using a suitable example. Also analyze the performance of the algorithm in the worst case.

(b) What is the time complexity for Strassen's matrix multiplication algorithm?

(c) Prove that sub-paths of shortest paths are also shortest.

(6 + 3) + 1 + 2 = 12

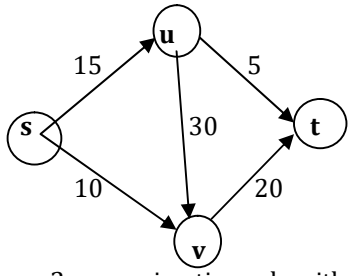
Group – E

8. (a) What is prefix function in the context of Knuth-Morris-Pratt (KMP) algorithm for pattern matching? Give an example to illustrate how it helps to reduce the time complexity of KMP algorithm.

(b) Draw a state-transition diagram for a string-matching automaton for the following pattern P over alphabet $\Sigma = \{a,b\}$.
$P = abbabaabbabbab$.

5 + 7 = 12

9. (a) Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represents capacity of the respective edge.



(b) Give a polynomial-time 2-approximation algorithm for vertex cover problem and also prove that the algorithm indeed achieves that factor.

(c) Just state whether the problems are in P or have been proved to be NP-complete. No explanation required.
(i) Hamiltonian Cycle problem
(ii) Edge Cover problem
(iii) 2-SAT Problem
(iv) Eulerian Cycle Problem

4 + (3 + 3) + 2 = 12