2015

Theory of Computation
(CSEN 5201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 x 1=10**

(i) A non-deterministic finite state machine (ndfsa) M having m states has been converted into a deterministic finite state machine (dfsa) N having n states. It follows that

(a) m = n (b) m < n
(c) m > n (d) m can be less than, equal to or greater than n.

(ii) There exists a deterministic finite state machine (dfsa) with n states that accepts all the strings in the regular expression 0*10*10*1 (and none others), where n equals

(a) 4 (b) 3 (c) 5 (d) 2.

(iii) L_1 and L_2 are two Type 3 (regular) languages. Let the language L consist of all strings that are in L_1 but not in L_2. Then

(a) L is a Type 3 language
(b) L is a Type 2 language but not necessarily a Type 3 language
(c) L is not necessarily a Type 2 language
(d) L is a Type 0 language but not necessarily a Type 1 language.

(iv) If the set of terminal symbols is { 0, 1, 2, 3 }, the total number of non-null strings of terminals of length at most 4 equals

(a) 256 (b) 320 (c) 336 (d) 340.

(v) Which one of the following languages cannot be accepted by a non-deterministic pushdown acceptor (ndpda)?

(a) $\{ 0^m1^n \mid 0 < m, 0 < n \}$ (b) $\{ 0^m1^n0^m \mid 0 < m, 0 < n \}$
(c) $\{ 0^m1^m0^m \mid 0 < m \}$ (d) $\{ 0^m1^n0^r \mid 0 < m, 0 < n, 0 < r \}$.

(vi) Which one of the following sets of positive integers is not a recursive set?

(a) { n | n is a perfect square }
(b) { n | n is not a prime number }
(c) { n | all planar graphs can be coloured properly using n colours }
(d) { n | n is not a prime number and n cannot be expressed as the sum of two or more prime numbers }.

(vii) A deterministic pushdown acceptor (dpda) can be constructed to recognize any given language L of Type n, where n equals

(a) 0 (b) 1 (c) 2 (d) 3.

(viii) The Pumping Lemma for Regular Languages can be used to prove that a given language L

(a) is not Type 0 (b) is not Type 1 (c) is not Type 2 (d) is not Type 3.

(ix) A machine M has been designed such that it accepts a given positive integer n as input, if and only if n is a prime number. Then which one of the following alternatives is false?

(a) M cannot be a deterministic finite state acceptor (dfsa)
(b) M cannot be a non-deterministic finite state acceptor (ndfsa)
(c) M cannot be a non-deterministic pushdown acceptor (ndpda)
(d) M cannot be a Turing machine.

(x) We are supplied two deterministic finite state acceptors (dfsas) M_1 and M_2 on the input alphabet {0,1}. We want to determine whether the languages $L(M_1)$ and $L(M_2)$ are identical sets, i.e., whether they contain exactly the same strings. Which of the following alternatives is true?

(a) There is a dfsa that will solve the problem
(b) There is a pda that will solve the problem but not a dfsa
(c) There is a Turing machine that will solve the problem but not a pda
(d) There is no effective procedure for solving the problem.

Group – B

2.(a) Design a deterministic finite state acceptor (dfsa) M_1 that will accept only those strings on the alphabet {0,1} which do *not* begin with the substring '010'. Show both the state table and the state transition diagram of M_1 and briefly explain how M_1 works.

(b) Design a deterministic finite state acceptor (dfsa) M_2 that will accept only those strings on the alphabet {0,1,2} in which the symbol 2 never follows the symbol 0. For example, M_2 will accept the strings 101200 and 10101 but not the string 110020. Show both the state table and the state transition diagram of M_2.

6+6=12

3.(a) A non-deterministic finite state acceptor (ndfsa) M_3 has the state table shown below. The start state is S and the only final state is C. Convert M_3 to an equivalent deterministic finite state acceptor (dfsa) M_4, clearly indicating the start and final states. Briefly explain your method of conversion.

	0	**1**
→ S	**S, A**	**S**
A	**--**	**B**
B	**--**	**C**
***C**	**C**	**C**

(b) Minimize the number of states in the machine M_4 designed above to get a dfsa M_5. Briefly explain the method of minimization.

6+6=12

Group – C

4.(a) Construct a deterministic finite state acceptor (dfsa) M_6 on the input alphabet { 0,1 } that accepts a string α if and only if α is contained in the regular expression 0*(0+1)(0+1)*.

(b) Construct a deterministic finite state acceptor (dfsa) M_7 on the input alphabet { 0,1 } that accepts a string α if and only if α is *not* contained in the regular expression 0*1*01.

6+6=12

5.(a) State and explain the Pumping Lemma for Regular Languages. Is it possible to use this lemma to show that a given language L is *not* context-free?

(b) Use the Pumping Lemma for Regular Languages to show that the language $L=\{ 0^m1^n0^m1^n \mid m > 5, n > 10 \}$ is not regular.

6+6=12

Group – D

6.(a) When is a Type 2 (context-free) grammar said to be ambiguous?Consider the grammar G = (V, T, S, P) where V = { S, A, B, 0, 1, 2, 3, 4 }, T = { 0, 1, 2, 3, 4 }, S is the start symbol, and the productions are as follows: S → A, S → B, S → 4, A → 012S, B → 012S3S. Show that the grammar G is ambiguous.

(b) Provide a Type 2 (context-free) grammar for the following language L over the input alphabet { 0, 1 }: L = { α | the string α has more 0's than 1's }

6+6=12

7.(a) Let G be a Type 2 (context-free) grammar in Chomsky Normal Form. Let α be a string in L(G) of length n, i.e., α has n terminal symbols. Explain why the parse tree of α must have exactly (2n -1) internal nodes (i.e., nodes with children).

(b) Using the Pumping Lemma for Context-Free Languages, show that the language $L=\{ 0^a1^b0^a1^b \mid a > 0, b > 0 \}$ is not a Type 2 (context-free) language.

6+6=12

Group – E

8.(a) Two positive integers m and n are written on a Turing machine tape in unary notation. The integer m is to the left of the integer n on the tape, and a single blank cell separates the two integers. At start the read/write head is positioned on the leftmost 1 of m. Give the state transition diagram of a Turing machine that will halt on the leftmost 1 of m if m >= n, and will halt on the leftmost 1 of n if m < n. Clearly state any assumptions made.

(b) When is a set of integers said to be a recursive set? What are the major characteristics of a recursive set? Explain whether the set of all positive integers that are not prime numbers is a recursive set.

6+6=12

9.(a) Briefly clarify what is meant by a 'Universal Turing Machine'. Can every effective procedure be implemented by a Universal Turing Machine assuming the input is appropriately supplied?

(b) What is the 'Halting Problem' for Turing Machines? "Halting Problem for Turing machines is unsolvable" - comment.

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