Group - D

6. (a) Test whether the set of vectors { (1,2,1) , (2,1,1) , (1,1,2) } are linearly dependent.

   (b) Find a basis and dimension of the subspace W of R^3 where $W = \{(a,b,c) \; : \; a+b+c=0\}$.

   (c) Find the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{pmatrix}$

3 + 6 + 3 = 12

7. (a) Prove that the intersection of any two subspaces W_1 and W_2 of a vector space V(F) is also a subspace of V(F).

   (b) Let $G : R^3 \rightarrow R^3$ be the linear mapping defined by $G(x,y,z) = (x+2y-z, y+z, x+y-2z)$. Find a basis and the dimension of (i) image of G (ii) Kernel of G.

5 + 7 = 12

Group - E

8. (a) A firm has a total revenue function, $R(x) = 20x - 2x^2$, and a total cost function, $C(x) = x^2 - 4x + 20$, where x represents the quantity. Find the revenue maximizing output level and the corresponding value of profit.

   (b) Solve the following problem by Lagrange multiplier method

$$Min\ z = x_1^2 + x_2^2 + x_3^2$$

Subject to the constraints

$$x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$

4 + 8 = 12

9. (a) Solve the following $L.P.P.$ by using the simplex method

$$\text{Maximize } z = 2x_1 + 3x_2$$

Subject to the constraints

$$x_1 + x_2 \leq 1$$
$$3x_1 + x_2 \leq 4$$
$$x_1, \; x_2 \geq 0$$

(b) Find the maximum and minimum value of the function

$$f(x_1, x_2) = 20x_1 + 26x_2 + 4x_1x_2 - 4x_1^2 - 3x_2^2 .$$

8 + 4 = 12

M.TECH/ECE/VLSI/1st SEM/MATH 5103/2017
ADVANCED MATHEMATICS
(MATH 5103)

Time Allotted: 3 hrs **Full Marks: 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1=10**

   (i) If the feasible set of an optimization problem is unbounded, then
   (a) no finite optimum point exists
   (b) it has an infinite number of feasible points
   (c) the existence of a finite optimum point cannot be assured
   (d) none of these.

   (ii) The function $f(x,y) = x^2y + y^2 + 2y$ has
   (a) a maximum point (b) a minimum point
   (c) a saddle point (d) none of these.

   (iii) A fair dice is thrown. The probability that either an odd number or a number greater than 4 will turn up is
   (a) 2/5 (b) 3/7 (c) 2/7 (d) 2/3.

   (iv) A connected planar graph with 5 vertices determines 3 regions. The number of edges of the graph is
   (a) 6 (b) 5
   (c) 2 (d) this type of graph does not exist.

   (v) If C_{97} be a circuit with 97 number of vertices then $\chi(C_{97}) =$
   (a) 97 (b) 98 (c) 2 (d) 3

   (vi) Independence number for a complete bipartite graph $K_{m,n}$ is
   (a) min (m, n) (b) $m+n$
   (c) max(m, n) (d) n

(vii) Possible transition probability matrix of Markov chain is

(i) $\begin{pmatrix} 0.2 & 0.6 \\ 0.5 & 0.5 \end{pmatrix}$ (ii) $\begin{pmatrix} 0.1 & 0.9 \\ 1 & 0 \end{pmatrix}$

(a) (i) only (b) (ii) only
(c) (i) and (ii) both (d) none of these

(viii) If X has a Poisson distribution with parameter μ, then the mean of X is

(a) μ (b) μ^2 (c) $\mu(\mu-1)$ (d) 1

(ix) The characteristic equation of the matrix A is $t^2-t-1=0$, then

(a) A^{-1} does not exit (b) $A^{-1}=I$
(c) $A^{-1}=A+I$ (d) $A^{-1}=A-I$

(x) The sum of eigen values of $\begin{pmatrix} -1 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & -3 \end{pmatrix}$ is

(a) -3 (b) -1 (c) 3 (d) 1

Group – B

2. (a) Three machines X, Y, Z produce respectively 60%, 30% and 10% of the total number of items of a factory. Of this output 2% ,3% and 4% are defective. An item is selected at random and is found defective. Find the probability that the item was produced by machine Z.

(b) A random variable X has the following probability function

$X=x$:	- 2	- 1	0	1	2	3
$P(X=x)$:	0.1	k	0.2	2k	0.3	3k

(i) Find k,
(ii) Find the distribution function
(iii) Find $P(-2 \le X \le 2)$

6 + 6 = 12

3. (a) Suppose the mean of a binomial distribution is 3 and the variance is 3/2. Find the probability of obtaining at most 3 successes.

(b) Three children (denoted by 1, 2, 3), arranged in a circle, play a game of throwing a ball to one another. At each stage the child having the ball

M.TECH/ECE/VLSI/1st SEM/MATH 5103/2017

is equally likely to throw it into any one of the other two chi
Suppose that X_0 denotes the child who had the ball initiall
X_n $\{n \ge 1\}$ denotes the child who had the ball after n throws.
$\{X_n, n \ge 1\}$ forms a Markov chain. Find corresponding tran
probability matrix. Also calculate
$P(X_2=1 \mid X_0=1),\quad P(X_2=2 \mid X_0=3) \quad and \quad P(X_2=3 \mid X_0$

5 + 7

Group - C

4. (a) If G be a simple connected planar graph with $n \ge 3$ vertices, edges and f faces, prove that (i) $e \ge \frac{3}{2} f$ (ii) $e \le 3n-6$.

(b) Consider a connected, simple planar graph with 20 vertices, e degree 3. Into how many regions does a representation of this graph divide the plane?

(c) Show that a simple connected planar graph has a vertex of degree 5 o

6 + 3 + 3

5. (a) Using Decomposition theorem, determine the chromatic polynon the graph of fig.1.

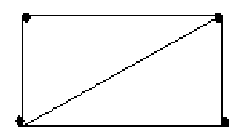

Fig.1

Hence find its chromatic number.

(b) Find the maximal matching, maximum matching fig.2 and all po maximal matching for fig.3. Find the matching numbers for both f fig.3. Also find if there exists a perfect matching for both the grap

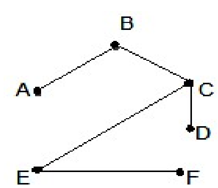

Fig.2

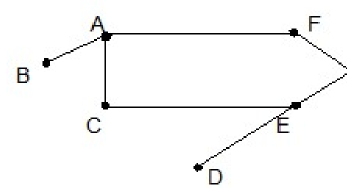

Fig.3

6 + 3 + 3