M.TECH/AEIE/1st SEM/MATH 5101/2017
ADVANCED MATHEMATICAL METHODS
(MATH 5101)

Time Allotted: 3 hrs **Full Marks: 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1=10**

(i) If V and W are finite dimensional vector spaces over a field F and $T: V \longrightarrow W$ is a linear mapping, then
(a) Rank of T + Nullity of T = dim V
(b) Rank of T + Nullity of T = dim W
(c) Rank of T + Nullity of T > dim V
(d) None of these.

(ii) If $\lambda^3 - 6\lambda^2 + 9\lambda = 4$ is the characteristic equation of a square matrix A then A^{-1} is equal to
(a) $A^2 - 6A + 9I$ (b) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}I$
(c) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}$ (d) $A^2 - 6A + 9$.

(iii) Let the linear transformation $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be $T(x, y) = (x, x + y, y)$. Then dim Ker T is
(a) 0 (b) 1
(c) 2 (d) 3.

(iv) Consider the polynomials $f(t) = 3t - 2$ and $g(t) = t + 2$ in $P(t)$ with the inner product $< x, y > = \int_0^1 x(t)y(t)dt$
Then $< f, g >$ is.
(a) -1 (b) 1
(c) 0 (d) 2.

(v) The number of linearly independent eigenvectors of $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 5 \end{pmatrix}$ is
(a) 1 (b) 2
(c) 3 (d) 4.

(vi) If in the simplex algorithm, the set of basic variables of the final simplex table contains an artificial variable, the problem has
(a) degenerate solution (b) infeasible solution
(c) unbounded solution (d) multiple solution.

(vii) For a general non-linear programming problem Kuhn-Tucker conditions are
(a) sufficient (b) necessary
(c) necessary and sufficient (d) not applicable.

(viii) If (a, b) is a stationary point of the function $f(x, y)$ such that $\frac{\partial^2 f(a,b)}{\partial x^2} = 2$ and the determinant of the Hessian matrix of the function $f(x, y)$ at (a, b) i.e. det $(H(f(a, b))) = 3$ then
(a) (a, b) is a local maximum point
(b) (a, b) is a local minimum point
(c) (a, b) is a saddle point
(d) none of these.

(ix) The number of stationary points of the function $f(x, y) = x^2 + 3y$ is
(a) 1 (b) 2 (c) 3 (d) 4.

(x) The dual of a dual is
(a) dual (b) primal
(c) neither primal nor dual (d) none of these.

Group - B

2. (a) Examine if the set S is a subspace of $\mathbb{R}^3$, where $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$.

(b) Show that if the vectors $\{\alpha, \beta, \gamma\}$ are linearly independent, then the vectors $\{\alpha + \beta,\ \alpha - \beta,\ \alpha - 2\beta + \gamma\}$ are also linearly independent.

6 + 6 = 12

3. (a) Examine if the set S is a subspace of $\mathbb{R}^3$, where $S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$.

(b) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace U of $\mathbb{R}^4$ spanned by $S = \{(1,1,1,1), (1,1,2,4), (1,2,-4,-3)\}$.

6 + 6 = 12

Group - C

4. (a) Find all eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$.

(b) Prove that the characteristic equation of an orthogonal matrix A is a reciprocal equation.

6 + 6 = 12

5. (a) Suppose the mapping $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is defined by $T(x, y) = (x + y, x)$. Show that T is linear.

(b) Determine the linear mapping $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ which maps the basis vectors $\{(0,1,1), (1,0,1), (1,1,0)\}$ to $\{(1,1,1), (1,1,1), (1,1,1)\}$. Verify $dim\ Ker\ T + dim\ Im\ T = 3$.

6 + 6 = 12

Group - D

6. (a) Solve the following problem by Lagrange multiplier method

Minimize $z = (x-3)^2 + (y+1)^2 + (z-2)^2$

Subject to the constraints

$$3x - 2y + 4z = 9$$
$$x + 2y = 3$$

(b) Find the maximum and minimum value of the function

$$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$

7 + 5 = 12

7. Maximize $f(x_1, x_2) = 10x_1 - x_1^2 + 10x_2 - x_2^2$

Subject to the constraints

$$x_1 + x_2 \leq 9$$
$$x_1 - x_2 \geq 6$$
$$x_1, x_2 \geq 0$$

by applying Kuhn-Tucker conditions

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Group - E

8. (a) Apply simplex method to solve the following L.P.P :

Maximize $z = 30x_1 + 23x_2 + 29x_3$

Subject to

$$6x_1 + 5x_2 + 3x_3 \leq 26$$
$$4x_1 + 2x_2 + 5x_3 \leq 7$$
$$x_1, x_2, x_3 \geq 0$$

From the final table find the optimal solution of the dual problem.

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9. Solve by Charne's Big M –method, the following L.P.P.

Maximize $Z = 4x_1 + 2x_2$

Subject to

$$3x_1 + x_2 \geq 27$$
$$x_1 + x_2 \geq 21$$
$$x_1 + 2x_2 \geq 30$$
$$x_1, x_2 \geq 0$$

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