B.TECH /AEIE/ ECE/EE/CE/3RD SEM/ MATH 2001/2017

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of $\oint_C \frac{dz}{z-4}$ where C is the circle $|z-1| = 2$ is
(a) $2\pi i$ (b) 0 (c) πi (d) $4\pi i$.

(ii) Which of the following functions has an essential singularity at $z = 0$?
(a) $\frac{1}{z}$ (b) $\frac{1}{z} + \frac{1}{z^2}$ (c) e^{z^2} (d) $e^{\frac{1}{z^3}}$.

(iii) Let C be a closed curve enclosing the origin. Then $\oint_C \frac{\cos z}{z} dz$ is
(a) πi (b) $2\pi i$ (c) $-\pi i$ (d) $4\pi i$.

(iv) The Fourier sine transform of $f(x) = e^{-ax}, a > 0$ is
(a) $\frac{s}{(a^2+s^2)}$ (b) $\frac{a}{(a^2+s^2)}$ (c) $\frac{2s}{(a^2+s^2)}$ (d) none of these.

(v) $\int_{-1}^{1} P_0(x)dx =$
(a) 4 (b) 1 (c) 2 (d) 0.

(vi) The period of the function $f(x) = |\sin x|$ is
(a) 2π (b) $\frac{\pi}{2}$ (c) π (d) 1.

(vii) For the differential equation
$x^2(1-x)y'' + xy' + y = 0$
(a) $x = 1$ is an ordinary point
(b) $x = 1$ is a regular singular point
(c) $x = 1$ is an irregular singular point
(d) $x = 0$ is an ordinary point.

(viii) $\frac{d}{dx}\{x^4 J_4(x)\} =$
(a) $x^4 J_3(x)$ (b) $x^3 J_3(x)$ (c) $x^4 J_5(x)$ (d) $-x^4 J_3(x)$.

(ix) The P.I of $(D^2 - D'^2)z = \cos(x+y)$ is
(a) $\frac{x}{2}\cos(x+y)$ (b) $\frac{x}{2}\sin(x+y)$
(c) $x\sin(x+y)$ (d) $x\cos(x+y)$.

(x) The value of $\lim_{z\to i}\frac{iz+1}{z-i}$ is
(a) 1 (b) -1 (c) i (d) -i.

Group – B

2. (a) Find the imaginary part $v(x,y)$ of the analytic function $f(z) = u(x,y) + iv(x,y)$ where $u(x,y) = e^{2x}(x\cos 2y - y\sin 2y)$.

(b) Show that $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\,co}\,d\theta = \frac{\pi}{12}$

6 + 6 = 12

3. (a) Evaluate $\oint_C \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}dz$ where C is the circle $|z| = 3$.

(b) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\oint_C f(z)dz$, where C is the circle $|z| = 2.5$.

6 + 6 = 12

Group – C

4. (a) Find the Fourier transform of the function $f(x) = \begin{cases}1, \text{ for } |x| < 1\\ 0, \text{ for } |x| > 1\end{cases}$
Hence evaluate $\int_0^{\infty}\frac{\sin x\cos\lambda x}{x}dx$.

(b) Using Parseval's identity for the following function
$f(x) = \begin{cases}-x, \text{ for } -2 \le x \le 0\\ x, \text{ for } 0 \le x \le 2\end{cases}$
prove that $\sum\frac{1}{n^4} = \frac{\pi^4}{96}$, $n = 1,3,5......$. **6 + 6 = 12**

5. (a) Evaluate $F^{-1}[e^{-a|s|}], a > 0$.

(b) Find $f(x)$ if its Fourier cosine transform is
$\bar{f}_c(s) = \begin{cases}a - \frac{s}{2}, & \text{for } s < 2a\\ 0, & \text{for } s \ge 2a\end{cases}$

6 + 6 = 12

Group – D

6. (a) Find the series solution of the differential equation

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-xy=0$$

about the point $x=0$.

(b) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

7 + 5 = 12

7. (a) Show that $J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$ where $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ and $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.

(b) Determine the singular points and check if they are regular or irregular:

$$x^2(x+1)^2y''+(x^2-1)y'+2xy=0.$$

(c) Solve by finite difference method: $y''(x)+y(x)+1=0$, where $y(0)=0,\ y(1)=0$ and $h=0.5$.

4 + 2 + 6 = 12

Group – E

8. (a) Form the partial differential equation (by eliminating the arbitrary function) from the relation $z=(x+y)\emptyset(x^2-y^2)$.

(b) Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$

6 + 6 = 12

9. (a) Find the general solution of the following partial differential equation

$$\frac{\partial^2 z}{\partial x^2}-7\frac{\partial^2 z}{\partial x\partial y}+12\frac{\partial^2 z}{\partial y^2}=e^{x-y}.$$

(b) Solve the heat equation given by

$$\frac{\partial u}{\partial t}=\frac{1}{h^2}\frac{\partial^2 u}{\partial x^2}$$

by the method of separation of variables.

$\left(\text{Given that } u(0,t)=0, u(l,t)=0, u(x,0)=\sin\frac{\pi x}{l}\right)$.

5 + 7 = 12