DESIGN AND ANALYSIS OF ALGRORITHMS
(MCAP 2101)

Time Allotted: 3 hrs **Full Marks: 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1= 10**

(i) Ω-notation provides an asymptotic
(a) upper bound (b) lower bound
(c) loose bound (d) none of these.

(ii) The minimum number of colours needed to colour a graph having n > 3 vertices and 2 edges is
(a) 2 (b) 3 (c) 4 (d) 1.

(iii) Kruskal's algorithm for finding Minimal Spanning Tree of a Graph is a
(a) divide & conquer algorithm
(b) branch and bound algorithm
(c) greedy algorithm
(d) dynamic programming algorithm.

(iv) The node removal of which makes a graph disconnected is called
(a) pendant vertex (b) bridge
(c) articulation point (d) coloured vertex.

(v) Time complexity of non-deterministic algorithm is always
(a) less than deterministic algorithm
(b) greater than deterministic algorithm
(c) equal to deterministic algorithm
(d) none of these.

(vi) Breadth-First-Search on a graph G = (V, E) has running time
(a) O (|V| + |E|) (b) O (|V|)
(c) O (|E|) (d) none of these.

(vii) 'Small o' of g (n) is
(a) asymptotically loose (b) asymptotically tight
(c) same as 'Big O' (d) none of these.

(viii) The running time of an algorithm on a particular input is measured by
(a) counting the running time in microseconds
(b) counting the number of primitive operations
(c) counting the number of statements
(d) none of these.

(ix) The time-complexity of Traveling Salesman Problem is
(a) $O(n^2\, 2^n)$ (b) $\Theta(n^2\, 2^n)$
(c) $\Omega(n^2\, 2^n)$ (d) none of these.

(x) Which of the following is true
(a) all NP hard problems are NP complete
(b) some NP complete problems are NP hard
(c) all NP complete problems are NP hard
(d) none of these.

Group - B

2. (a) Give the analysis of Quick Sort with its Worst Case Scenario.

(b) Find asymptotic tight bounds of the following recurrence relations using Master Method:
(i) $T(n) = 4T(n/2) + n^3$ (ii) $T(n) = 2T(n/4) + \sqrt{n}$.

(c) Explain the difference between Big-O and Small-o asymptotic notations.

5 + 4 + 3 = 12

3. (a) Suppose Quick sort Algorithm is applied to sort the following input sequence.

<77, 8, 56, 3, 23, 11, 88, 7, 32, 20>

Considering first element as the pivot, show the partially sorted array after the initial pass when partition is carried out by
(i) Hoare method and (ii) Lomuto method.

(b) What is ω-notation stand for? Show that for any real constants a and b, where b > 0, $(n + a)^b = \Theta(n^b)$.

(c) Is merge sort a stable sorting method? Justify your answer.

(2 + 2) + (1 + 3) + 4 = 12

Group - C

4. (a) Write an algorithm for finding the minimal spanning tree of a graph. Discuss its time complexity.

   (b) Given the weight vector (2, 3, 5, 7, 1, 4, 1) and the profit vector (10, 5, 15, 7, 6, 18, 3) and a Knapsack of capacity 15, find an optimal solution for the fractional knapsack problem.

8 + 4 = 12

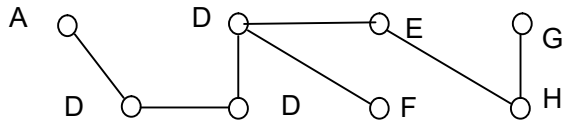
5. (a) Differentiate between divide-and-conquer and dynamic programming.

   (b) Discuss the Bellman-Ford's algorithm for single-source shortest path problem with an example. What is the time complexity of the algorithm?

4 + (6 + 2) = 12

Group - D

6. (a) Apply the Depth-First-Search (DFS) Algorithm to traverse the nodes of the following graph considering C as the starting vertex.



   (b) What is the utility of failure function in Knuth-Morris-Pratt pattern matching? Write an algorithm to calculate the failure function. Calculate the failure function for the pattern *aababaaba*.

4 + (1 + 5 + 2) = 12

7. (a) How the bidirectional search helps to reduce the complexity of the standard search algorithms? Write an algorithm to implement the same. What is the time complexity of this algorithm?

   (b) What is the chromatic number of a graph? Prove that a graph is 2 chromatic if and only if it is bipartite.

(2 + 3 + 2) + (1 + 4) = 12

Group - E

8. (a) Give an example of NP-hard decision problem which is not NP-complete. Give reasons for the same.

   (b) Resolve the 8 queens problem in terms of branch and bound strategy. Compare this approach with backtracking.

5 + (4 + 3) = 12

9. (a) What is meant by an approximation algorithm? How Approximation Ratio ρ(n) helps to find the performance of this type of algorithms?

   (b) Prove that the vertex cover problem (VCP) is NP-Complete.

(2 + 3) + 7 = 12