Group - E

8. (a) Briefly discuss with example type-0, type-1, type-2 and type-3 grammars.

(b) From the state transition table of the following NFA obtain it's equivalent DFA.

PS \ Σ	NS	
	a=0	a=1
→S_0	$\{S_0,S_1\}$	$\{S_1\}$
(S_1)	$\{S_2\}$	$\{S_2\}$
S_2	ϕ	$\{S_2\}$

(1.5 × 4) + 6 = 12

9. (a) Define Mealy machine and Moore machine. Construct a Moore machine from the following Mealy machine :

Present State	Next State			
	a=0		*a=1*	
	State	Output	State	Output
S_0	S_0	1	S_1	0
S_1	S_3	1	S_3	1
S_2	S_1	1	S_2	1
S_3	S_2	0	S_0	1

(b) Show that the language $\{0^n 10^n : n \geq 1\}$ is not a regular language.

4 + 5 + 3 = 12

DISCRETE MATHEMATICS
(MCAP 1104)

Time Allotted: 3 hrs **Full Marks: 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1=10**

(i) If the general term of the sequence $\{ a_k \} = a^k$,then the corresponding generating function is
(a) $1/(1 - ax)$ (b) $1/(1 - x)$ (c) $k/(1 - x)$ (d) $a/(1 - x)$.

(ii) If n be the number of vertices, e be the number of edges and k be the number of components then
(a) $e > n+k$ (b) $e \geq n-k$ (c) $e \leq n-k$ (d) none of these.

(iii) If the function $f : R \rightarrow R$ is defined by $f(x)= 3x - 4$, when $x > 0$
$= - 3x + 2$, when $x \leq 0$
then $f^{-1}(2) =$
(a) {2} (b) {0, 2} (c) {-2, 2} (d) none of these.

(iv) The cardinality of a power set of a non-empty set A is
(a) $2^{|A|}$ (b) $2|A|$ (c) $|A|^2$ (d) $|A|^2 - |A|$.

(v) In how many ways can a president and vice president be chosen from a set of 30 candidates?
(a) 820 (b) 850 (c) 880 (d) 870.

(vi) The proposition $P \wedge (\sim P \vee Q)$ is a
(a) Tautology
(b) logically equivalent to $P \wedge Q$
(c) logically equivalent to $P \vee Q$
(d) a contradiction.

(vii) What is the minimum number of vertices necessary for a graph with 6 edges?
(a) 6 (b) 5 (c) 7 (d) none of these.

(viii) A partial ordered relation is transitive, reflexive and
(a) antisymmetric (b) bisymmetric
(c) anti reflexive (d) asymmetric.

(ix) If N be the set of all natural numbers then the mapping $f : N \to N$ defined by

$$f(n)=\begin{cases} 2n, \text{ if n is even} \\ n, \text{ if n is odd} \end{cases}$$ is

(a) onto (b) one-to-one
(c) both (a) and (b) (d) none.

(x) Which of the following propositions is a tautology?
(a) $(p \vee q) \rightarrow q$ (b) $p \vee (q \rightarrow p)$
(c) $p \vee (p \rightarrow q)$ (d) both (b) & (c).

Group - B

2. (a) Prove that for any three sets A, B, C : $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b) A relation ρ is defined on the set N of natural numbers such that **"m ρ n iff m is a divisor of n" ∀ m, n ∈ N.**
Examine if ρ is
(i) reflexive (ii) symmetric (iii) transitive.

(c) Prove that $(P \wedge (P \leftrightarrow Q)) \rightarrow Q$ is a tautology.

4 + (2 + 2 + 2) + 2 = 12

3. (a) Consider $f : Z^+ \to Z^+$ defined by $f(a) = a^2$. Is *f* one-to-one? Is *f* onto? Explain.

(b) Let $f : A \to B$, $g : B \to C$ and $h : B \to C$ be mappings such that $(g \circ f) = (h \circ f)$ and *f* is surjective. Prove that $g = h$.

(c) Prove that $p \vee (\sim p \wedge q) \equiv (p \vee q)$

4 + 4 + 4 = 12

Group – C

4. (a) Find the generating function for the sequence {7, 8, 9, 10,.....}

(b) Find the coefficient of x^{18} in
$(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + x^5 + \cdots)^5$

(c) Find the general solution of the following recurrence relation
$a_{n+2} - 5a_{n+1} + 6a_n = 4^n, n \geq 0$

4 + 4 +

5. (a) What is the number of permutations in the letters of the MISSISSIPPI where 4S's do not come together?

(b) Show that if any 30 people are selected, then we can choose a sub 5 so that all 5 were born on the same day of the week.

(c) Consider a set of integers from 1 to 250. Find how many of numbers are divisible by 3 or 7 or 5. Also indicate how mar divisible by 3 or 7 but not by 5 and divisible by 3 or 5.

3 + 3 +(2 + 2 + 2

Group - D

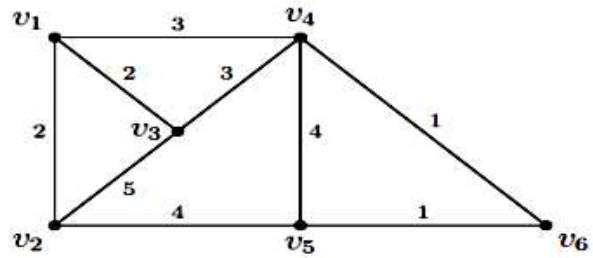
6. (a) Prove that the number of vertices of odd-degree in a graph G is a even.

(b) Prove that the maximum number of edges in a graph G with n ve and k components is (n - k)(n - k +1)/2. From this expression fi maximum number of edges in a graph G with only one componen

5 + (6 + 1

7. (a) Draw a binary tree to represent the mathematical expression
$(a - b)/(c * (d - e))$.

(b) What is a Minimal Spanning Tree? Find a minimal spanning tree Kruskal's algorithm for the following weighted graph wher numbers represent the weight of the corresponding edge. What total weight of the minimal spanning tree? Also calculat complexity of the algorithm.



4 + (2 + 4 + 2