NUMERICAL AND STATISTICAL TECHNIQUES
(MCAP 1103)

Time Allotted: 3 hrs **Full Marks: 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
Any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1=10**

(i) There are 8 red, 7 blue and 6 green balls in a box. One ball is picked up randomly. What is the probability that it is neither blue nor green?
(a) $^2/_3$ (b) $^8/_{21}$ (c) $^9/_{22}$ (d) $^3/_7$.

(ii) For the p.d.f.
$f(x) = (^1/_4), -2 < x < 2$
$= 0,$ otherwise
Then the value of E(X) is
(a) 1 (b) -1 (c) 0 (d) 3.

(iii) The mean of the Binomial distribution b(10, 2/5) is
(a) 4 (b) 6 (c) 5 (d) 0.

(iv) If $\hat{\theta}$ is the estimator of the parameter θ, then $\hat{\theta}$ is called unbiased if
(a) $E(\hat{\theta}) > \theta$ (b) $E(\hat{\theta}) < \theta$ (c) $E(\hat{\theta}) = \theta$(d) (iv) $E(\hat{\theta}) \neq \theta$.

(v) A passing student is failed by an examiner, it is an example of
(a) type-I error (b) type-II error
(c) best decision (d) all of the above.

(vi) If $f(x)$ is continuous in the interval *(a, b)* and if $f(a)$ and $f(b)$ are opposite signs, then there is
(a) at least one root of $f(x) = 0$ between a and b
(b) at most one root of $f(x) = 0$ between a and b
(c) there is no real root of $f(x) = 0$ between a and b
(d) none of these.

(vii) E^{-1} is equivalent to
(a) $1-\Delta$ (b) $1+\Delta$
(c) 1 (d) None of these.

(viii) If *f(x)* is a polynomial of degree n, then $\Delta^n f(x)$ is
(a) 0 (b) constant
(c) 1 (d) none of these.

(ix) Method of bisection is
(a) conditionally convergent (b) always convergent
(c) non-convergent (d) none of these.

(x) When a Guess elimination method is used to solve *AX = B, A* is transformed to a
(a) unit matrix (b) lower triangular matrix
(c) diagonal matrix (d) upper triangular matrix.

Group – B

2. (a) (i) If the events A_1 and A_2 are such that $P(A_1) \neq 0$ and $P(A_2) \neq 0$ and A_1 is independent of A_2, then prove that A_2 is independent of A_1.
(ii) If A_1 and A_2 are independent events, then so are A_1^c and A_2^c.

(b) A discrete random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) find k,
(ii) evaluate $P(0 < X < 5)$
(iii) $P(X \geq 5)$.

(3 + 3) + (2 + 2 + 2) = 12

3. (a) In a certain factory producing razor blades, there is a small chance, 1/500, for any blade to be defective. The blades are in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective and (iii) two defectives in a packet.

(b) State Chebyshev's Inequality and explain it's significance. What is the probability that the number of driving licences issued by Rod Transport Authority in a specific month is between 64 and 184, if the number of driving licences issued is a random variable with mean 124 and standard deviation 7.5?

(2 + 2 + 2) + (3 + 3) = 12

Group – C

4. (a) A random sample of size 1 is drawn from a Poisson population with parameter λ. Find an unbiased estimator of λ^2.

   (b) If T_1 and T_2 are statistics with $E(T_1) = \theta_1 + \theta_2$ and $E(T_2) = \theta_1 - \theta_2$, find unbiased estimator of θ_1 and θ_2.

6 + 6 = 12

5. (a) Distinguish between the following, in context of hypothesis testing: (i) test statistic and (ii) critical region.

   (b) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of perfectness of the coin is rejected if more than 4 heads are obtained.
   (i) What is the probability of Type I error?
   (ii) Find the probability of Type II error when the corresponding probability of head is 0.2.

(4 + 2) + (3 + 3) = 12

Group – D

6. (a) Using Lagrange's method of interpolation, find the polynomial *P(x)* of degree 2 such that :
   P(1)=1, P(3)=27, P(4)=64.

   (b) The speed, **v** of a car, **t** seconds after it starts, is shown in the following table:

t(s)	**0**	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
v(m/s)	**0**	**3.6**	**10.08**	**18.9**	**21.9**	**18.54**	**10.26**	**5.40**	**4.5**	**5.4**	**9**

   Using Simpson's 1/3rd rule, find the distance travelled by the car in 2 minutes.

   (c) Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors.

5 + 5 + 2 = 12

7. (a) Suppose 1.414 is used as an approximation to $\sqrt{2}$. Find the absolute and relative errors.

   (b) Apply Newton's backward interpolation to find y = *f(x)* at x = 4.12 from the following table :

x	**0**	**1**	**2**	**3**	**4**	**5**
y	**1**	**2**	**4**	**8**	**16**	**32**

   (c) Using Simpson's 1/3rd rule evaluate $\int_1^2 e^{\frac{-x}{2}}$, by dividing the range into 6 equal parts.

2 + 6 + 4 = 12

Group – E

8. (a) Evaluate y(0.1) using Euler's method when $\frac{dy}{dx} = 1 - y$, y(0) = 0 taking h = 0.01.

   (b) Find the real root of $3x - e^x = 0$ using bisection method.

   (c) Evaluate y(0.2) using Runge-kutta method of second order when $\frac{dy}{dx} = y - x$, y(0) = 2 taking h = 0.1.

3 + 5 + 4 = 12

9. (a) Using Bisection Method, find the real root of the equation $f(x) = x^3 - 3x - 5 = 0$, upto 2 decimal places.

   (b) Solve the following system of equations using the Gauss elimination method:
   $10x + 2y - 3z = 19,$
   $3x + 10y + 2z = 18,$
   $x + y + 10z = 13 .$

   (c) Use Regula-Falsi method to find the positive root of $xe^x = \cos x$, correct upto 2 decimal places.

4 + 4 + 4 = 12