ADVANCED CONTROL SYSTEM
(ELEC 4162)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For a linear time invariant system, the input signal with frequency $'f'$ affects the control error or output signal at the
(a) double frequency of $'f'$ (b) even multiple of $'f'$
(c) same frequency of $'f'$ (d) 3 times frequency of $'f'$.

(ii) Describing function of a multi-valued non-linear static element can be obtained by using
(a) fourier transform (b) laplace transform
(c) loop transformation (d) fourier series analysis.

(iii) A multi-valued nonlinear static element is excited with a pure sinusoidal signal $u_c(t) = 4\, units$ and frequency ω. The output signal is observed as $u(t) = 2\sin\omega t + 5\cos\omega t + 0.2\sin 2\omega t + 0.5\cos 2\omega t$. The describing function of the element is
(a) $0.50.5 < 0^0$ (b) $1.34 < 68.2^0$
(c) $1.25 < 0^0$ (d) $5.38 < 68.2^0$

(iv) The eigenvalues of a linearized system matrix $'A'$ of a second-order nonlinear system around a singular point are $-2 \pm 3j$. The resulting phase-plane trajectory of non-linear system is termed as
(a) stable node (b) stable focus
(c) center (d) saddle point.

(v) If the plot $\frac{-1}{N(A,,\omega)}$ crosses the Nyquist plot of $G(j\omega)$ from the inside of the encirclement to the outside of the encirclement as amplitude of sinusoidal input signal increases, the 'nonlinear system' exhibits
(a) stable limit cycle (b) unstable limit cycle
(c) non sinusoidal periodic signal (d) no limit cycle.

(vi) The phase-plane trajectory of 2nd order linear system converges to the origin along the isoclines as $t \to \infty$ when
(a) isocline slope approaches towards the fast eigenvector
(b) isocline slope = 0
(c) isocline slope = trajectory slope
(d) product of isocline slope and trajectory slope = -1.

(vii) The derivative of a 'Lyapunov energy function' of a 2nd order nonlinear system is given by $\dot{V}(x_1, x_2) = -4x_1^2 + 2x_1x_2 - 5{x_2}^2$, where x_1 and x_2 are state variables of the system. The nature of the stability of the system is
(a) stable in the sense of Lyapunov
(b) exponentially unstable system
(c) unstable
(d) asymptotically stable.

(viii) The quadratic scalar function
$f(x_1,x_2,x_3) = 2{x_1}^2 + 2x_1x_2 + 6x_1x_3 + 4{x_2}^2 + 8x_2x_3 + 10{x_3}^2$
for any values of $x_1 > 0, x_2 > 0$ and
(a) negative definite function (b) positive definite function
(c) positive semidefinite function (d) indefinite function.

(ix) A single-valued non-linear symmetric static element is excited with a sinusoidal signal of amplitude (A) and frequency (ω), describing function of the non-linear element can be determined when output signal is treated as
(a) $B\sin\omega t + A\cos\omega t$ (b) $B\sin 2\omega t$
(c) $B\sin\omega t$ (d) $B\sin 2\omega t + A\cos 2\omega t$.

(x) Definiteness of the matrix $Q = \begin{bmatrix} -20 & 0 & 0 \\ 0 & -5 & 1 \\ 0 & 1 & -4 \end{bmatrix}$ is
(a) positive definite (b) positive semidefinite
(c) negative definite (d) negative semidefinite.

Group – B

2. (a) Give a brief outline how to obtain the 'Describing Function(DF)' of a symmetric non-linear static function and also discuss the basic assumptions.

(b) Find the 'DF' of the non-linear static element shown in fig. 1 and plot $\frac{-1}{N(A)}\ vs\ A$.

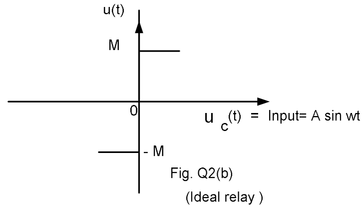

Fig. 1

5 + (5 + 2) = 12

3. (a) For the non-linear control system shown in fig. 2.

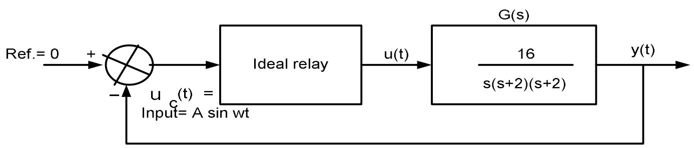

Fig.2

Investigate whether any limit cycle is present and if so determine its amplitude and frequency. Assume DF of the non-linear element is $N(A) = \frac{4}{\pi A}$.

(b) State and explain the different techniques that can be used to eliminate the' limit' cycle presents in a non-linear system.

8 + 4 = 12

Group – C

4. (a) A second-order non-linear system is described by $\ddot{x}(t) + 2\dot{x}^2(t)x(t) + x^2(t) - 2x(t) = 0$
   (i) Find the singular (equilibrium) points of the non-linear system and discuss the nature of the singular points.
   (ii) Roughly draw the nature of the trajectory around the singular point.

(b) Define the following terms:
   (i) Lyapunov stability
   (ii) Asymptotic stability
   (iii) Exponential stability
   (iv) Lyapunov energy function.

6 + 6 = 12

5. (a) Check the stability of the non-linear system $\dot{x}_1(t) = x_2(t)$ and $\dot{x}_2(t) = -x_1(t) - x_2^3(t)$ using 'Lyapunov' energy function method and it is assumed as $V(x_1(t), x_2(t)) = x_1^2(t) + x_2^2(t)$. Is the system 'asymptotically' stable?

(b) A linear system is described by the state equation $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t)$. Investigate the stability of this system using 'Lyapunov's equation with the choice of $Q = I_{2x2}$. Show that the solution of the 'Lyapunov equation is 'positive definite'?

5 + (6 + 1) = 12

Group – D

6. (a) Discuss briefly, the frequency response of a linear time invariant discrete time system to a sinusoidal input.

(b) Consider the system defined by $x(kT) = ax[(k-1)T] + u(kT)$, $0 < a < 1$. Obtain the steady state response $x_{ss}(kT)$ when input $u(kT)$ is the sampled sinusoidal signal i.e. $u(kT) = A\sin(\omega kT)$.

6 + 6 = 12

7. (a) Explain clearly the term "pulse-transfer function" of a sampled data system?

(b) Consider the unity feedback digital control system, the loop-transfer function is given as $GH(z) = \frac{0.0952K\,Z}{(z-1)(z-0.905)}$. Draw the root-locus of the closed loop digital control system. Find the range of K for which the system is stable.

3 + (7 + 2) = 12

Group – E

8. (a) For an observer based control system, show that the controller gain $'K'$ and observer gain $'L'$ can be designed independently.

(b) A system described by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

   (i) Check the observability of the system
   (ii) Design a full-order observer using either direct comparison method or 'Ackerman's 'method. Assume the observer eigenvalues are placed at $\lambda_{01} = -8$ and $\lambda_{02} = -10$ respectively.

6 + 6 = 12

9. (a) Discuss the significance of each "Quadratic term" that involves in the performance index $J(.)$ of "Linear Quadratic Regulator (LQR)" problem.

(b) Consider a first-order system $\dot{x}(t) = 2x(t) + 4u(t)$ with the cost function as $J(x(t)) = \frac{1}{2}\int_0^{\infty}(4x^2\,(t) + u^2(t))dt$

(i) Write the corresponding algebraic matrix Riccati equation (AMRE) and hence solve the AMRE.

(ii) Find the optimal control law $u(t) = -kx(t)$ and the draw the closed-loop system based on optimal control law.

5 + (5 + 2) = 12