B.TECH/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/1ST SEM/MATH 1101/2017

MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{pmatrix}$ is singular then $\lambda = ?$

(a) 0 (b) 4 (c) 2 (d) -1.

(ii) Rank of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is

(a) 0 (b) 1 (c) 3 (d) 2.

(iii) The series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

(a) converges (b) diverges
(c) oscillates finitely (d) oscillates infinitely.

(iv) $\int_0^{\pi/2} \sin^5 x dx =$

(a) $7/15$ (b) $8\pi/15$ (c) $8/15$ (d) $4/15$

(v) If $x = r\cos\theta, \quad y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is

(a) $-1/r$ (b) - r (c) r (d) $1/r$

(vi) If $y = \sin 3x + \cos 3x$, then $(y_n)_0$ is

(a) $(-1)^n 3^n$ (b) $\sin\frac{n\pi}{2} + \cos\frac{n\pi}{2}$ (c) 0 (d) 3^n

(vii) If $u = \frac{x^2y^4}{x^6+y^6}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$

(a) $6u$ (b) u (c) 0 (d) $3u$

(viii) The value of x which makes the vectors $x\hat{\imath} + 2\hat{\jmath} + 8\hat{k}$ and $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ mutually perpendicular is

(a) 0 (b) -1 (c) 1 (d) 2

(ix) The value of $\iint_R dx\,dy$ where R is the region enclosed by $x^2 + y^2 = 4$ is

(a) 1 (b) 2π (c) π (d) 4π

(x) The directional derivative of $\Psi = xy + yz + zx$ at the point (1, 1, 1) in the direction of positive z-axis is

(a) 1 (b) 2 (c) 3 (d) 4.

Group – B

2. (a) Solve the following system of linear equations by Cramer's rule:

$$x + y + z = 1$$
$$ax + by + cz = k$$
$$a^2x + b^2y + c^2z = k^2$$

where $a \neq b \neq c$.

(b) Use Laplace's method of expansion and prove

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & d & 0 & f \\ -c & e & -f & 0 \end{vmatrix} = (af - be + cd)^2$$

6 + 6 = 12

3. (a) Find the rank of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ for different values of b.

(b) Find whether the given system of equations is consistent or not with proper justifications:

$$x + 2y - z = 10$$
$$-x + y + 2z = 2$$
$$2x + y - 3z = 2$$

6 + 6 = 12

Group – C

4. (a) Show that the following series is conditionally convergent:

$$\frac{1}{2}-\frac{2}{5}+\frac{3}{10}-\frac{4}{17}+\frac{5}{26}-\dots\dots$$

(b) Is Rolle's theorem applicable for the following functions?

(i) $f(x)=\frac{1}{2-x^2}$ in [-1,1]

(ii) $g(x)=\cos\left(\frac{1}{x}\right)$ in [-1,1]

(Justify your answer in detail.)

6 + (3 + 3) = 12

5. (a) Verify Rolle's theorem for the given function:
$f(x)=x^2-5x+6 \quad 1\le x\le 4$

(b) Apply Lagrange's Mean Value Theorem to prove that $1+\frac{x}{2\sqrt{1+x}}<\sqrt{1+x}<1+\frac{x}{2}$ for -1 < x < 0

(c) Determine the behaviour of $\sum_{n=1}^{\infty}\sin\left(\frac{1}{\sqrt[3]{n}}\right)$.

6 + 3 + 3 = 12

Group – D

6. (a) If $y=\left(\frac{1}{\sqrt{1-x^2}}\right)\cos^{-1}x$, show that $(1-x^2)y_{n+1}-(2n+1)xy_n-n^2y_{n-1}=0$.

(b) If $y_1=\frac{x_2x_3}{x_1}, y_2=\frac{x_3x_1}{x_2}, y_3=\frac{x_1x_2}{x_3}$, find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3.

6 + 6 = 12

7. (a) If $f(x,y)=\begin{cases} xy\frac{4x^2+5y^2}{x^2+y^2} & ,(x,y)\neq(0,0) \\ 0 & ,(x,y)=(0,0)\end{cases}$

Prove that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

(b) If $y=\sin(m\sin^{-1}x)$, (m is a constant) prove that

(i) $(1-x^2)y_2-xy_1+m^2y=0$

(ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(x^2-m^2)y_n=0$.

6 + (3 + 3) = 12

Group – E

8. (a) Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by $y=x$ and the parabola $y=4x-x^2$.

(b) Evaluate $\int_\Gamma\{(xy^2+x)dx+xydy\}$, where Γ is a closed curve of the region bounded by $y=x^2$ and $y^2=x$.

6 + 6 = 12

9. (a) Using Stoke's theorem evaluate $\int_C\{(x+y)dx+(2x-z)dy+(y+z)dz\}$, where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0), (0, 0, 6).

(b) If $J_n=\int_0^{\pi/2}x^n\sin x\,dx \quad (n>1)$, show that $J_n=n\left(\frac{\pi}{2}\right)^{n-1}-n(n-1)J_{n-2}$. Hence evaluate $\int_0^{\pi/2}x^5\sin x\,dx$.

6 + 6 = 12