PHYSICS - II
(PHYS 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The Hamiltonian of a system is a function of
(a) $(q_j, \dot{q}_j, t)$ (b) $(q_j, \dot{p}_j, t)$ (c) (q_j, p_j, t) (d) $(\dot{q}_j, \dot{p}_j, t)$.

(ii) If $[\hat{\alpha}, \ \hat{\beta}] = 1$, then
(a) $[\hat{\alpha}, \ 2\hat{\beta}^2] = 2\hat{\beta}$ (b) $[\hat{\alpha}, 2\hat{\beta}^2] = 0$
(c) $[\hat{\alpha}, 2\hat{\beta}^2] = 4\hat{\beta}$ (d) $[\hat{\alpha}, 2\hat{\beta}^2] = 1$.

(iii) Which of the following function is eigen function of the operator d^2/dx^2
(a) $\psi = c \ln x$ (b) $\psi = c x^2$ (c) $\psi = c e^{-mx}$ (d) $\psi = c/x$.

(iv) The value of $[L^2, L_Z]$ is
(a) 1 (b) $i\hbar$ (c) $-i\hbar$ (d) 0.

(v) In a linear, isotropic dielectric, the relationship between the polarization vector $\vec{P}$, the electric susceptibility χ, and the electric field $\vec{E}$ is given by
(a) $\vec{P} = \chi\vec{E}/\varepsilon_0$ (b) $\vec{P} = \varepsilon_0\vec{E}/\chi$
(c) $\vec{P} = \chi\varepsilon_0\vec{E}$ (d) $\vec{P} = \chi\vec{E}$.

(vi) In an intrinsic semiconductor, the acceptor level
(a) lies near the valence band edge
(b) lies near the conduction band edge
(c) lies halfway between the valence and conduction band edges
(d) does not exist.

(vii) The density of states of free electrons in a metal varies with the energy E as
(a) E^2 (b) $\sqrt{E}$ (c) E (d) $\frac{1}{E}$

B.TECH/EE/IT/ME/3RD SEM/PHYS 2001/2017

(viii) If α_s and T be the space charge polarisability and temperature of a dielectric, then
(a) α_s increases with T (b) α_s decreases with T
(c) α_s is independent of T (d) α_s increases with T^2

(ix) The number of meaningful ways in which 4 fermions can be arranged in 3 compartments is
(a) 0 (b) 1 (c) 2 (d) 4.

(x) The waves representing a particle in an infinite square well are
(a) standing waves (b) progressive waves
(c) transverse waves (d) longitudinal waves.

Group – B

2. (a) What are generalized coordinates? Write down the Lagrangian and Lagrange's equation defining all the terms.

(b) What are the advantages of Lagrange's equation over the Newtonian equation of motion?

(c) The Lagrangian of a system is given by $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, where m and k are constants. Find the generalized momentum?

(d) Consider a particle sliding down an inclined plane. Write down Hamilton's equations for this system and solve them to find the equation of motion of the particle.

(1 + 1 + 1) + 2 + 2 + (2 + 3) = 12

3. (a) A system has two energy eigenstates ε_0 and $3\varepsilon_0$. ϕ_1 and ϕ_2 are the corresponding normalized eigenfunctions. At an instant the system is in a superposed state $\phi = C_1\phi_1 + C_2\phi_2$ and $C_1 = 1/\sqrt{2}$.
(i) Find the value of C_2, if ϕ is normalized.
(ii) What is the probability that an energy measurement would yield a value $3\varepsilon_0$.

(b) Find the eigenfunction of the momentum operator $\hat{P_x} = -i\hbar\frac{d}{dx}$, corresponding to the eigenvalue p

(c) Evaluate the commutator $[\hat{x}, \ \hat{L}_y]$.

(d) The spatial part of a wave function of a particle is given by,
$$\psi(x) = \begin{cases} x(x-1), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$
Calculate the probability of finding the particle in the region $0 \le x \le 0.5$.

(2 + 2) + 2 + 3 + 3 = 12

Group – C

4. (a) Write down the Fermi-Dirac distribution function, explaining all terms and define Fermi level in metal at absolute zero and at finite temperature.

(b) Using Fermi-Dirac statistics, calculate the concentration of holes in the valence band of an intrinsic semiconductor.

(c) 3 particles each of which can be in any one of the non-degenerate energy levels having energy values ε, 2ε, 3ε, 4ε. Find all possible macrostates of the particles in the energy levels for which the total energy of the system is 6ε. And also find the number of microstates of any one of the macrostes if
   (i) particles obeying B-E statistics.
   (ii) particles obeying F-D statistics.

(2 + 1 + 1) + 4 + (2 + 2) = 12

5. (a) Particles obeying M-B Statistics, write down the expression of thermodynamic probability of a macrostate (N_1, N_2, N_3,.......,N_i) having g_1, g_2, g_3,......, g_i be the number of energy states corresponding to 1st, 2nd, 3rd, ..., i th energy level respectively. From that expression establish $N(E)dE = \frac{g(E)dE}{e^{\alpha+\beta E}}$, where the symbols have their usual meaning.

(b) Find out the expression of average speed [v_{avg}] of ideal gas molecules. Given that $N(E)dE = \left[\frac{2\pi}{(\pi KT)^{\frac{3}{2}}}\right] e^{-\frac{E}{KT}} E^{\frac{1}{2}} dE$, where symbols have their usual meaning.

(c) Express the Fermi energy value in a metal in terms of free electron density at T = 0K.

(1 + 4) + 4 + 3 = 12

Group – D

6. (a) What is dielectric susceptibility? How is it related to the dielectric constant of dielectric material?

(b) Derive the expressions for the capacitance of a parallel plate capacitor (i) without dielectric and (ii) with a dielectric. (You may assume the expressions for the electric fields in each case.)

(c) The general expression of the average induced dipole moment in orientational polarization is $p = p_0\left[\cot h(a) - \frac{1}{a}\right]$, where $a = \frac{p_0 E}{KT}$, p_0, permanent dipole moment and E, electric field. Show that the orientational polarizibility is inversely proportional to the absolute temperature at high temperatures and weak electric fields.

(1 + 2) + (2 + 3) + 4 = 12

7. (a) Physically, what is a magnetic dipole and what is its magnetic dipole moment?

(b) The magnetic field intensity in a ferrite oxide is 10^6 A/m. If the susceptibility of the material at room temperature is 1.5×10^{-3}, compute the magnetization of the material and the magnetic field induction. ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2).

(c) State Curie's law of paramagnetism. Write down Weiss' hypotheses for a ferromagnetic material and derive the Curie-Weiss law.

(d) Draw the hysteresis loops of a soft and a hard magnetic material in the same plot.

(1 + 1) + (1½ + 1½) + (1 + 1 + 3) + 2 = 12

Group – E

8. (a) Show that when an electron moves through a crystal, the effective mass of the electron can be expressed as $m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$, where the symbols have their usual meaning.

(b) The energy – wave vector dispersion relation for a one-dimensional crystal of lattice constant a is given by $E(k) = E_0 - 2ak^2$ where E_0 is a constant. Find the expression of the effective mass of an electron in this crystal as a function of k.

(c) What is the critical magnetic field for a superconductor? How does it vary with temperature?

(d) Lead (Pb) gets transition to its superconducting state at 7.20 Kelvin. Lead has critical magnetic field strength at 0K is 65100 A/m, calculate its critical magnetic field strength at $-271^0\ C$.

(e) Distinguish between type I and type II superconductors. Name some materials belonging to these two types of superconductors.

2 + 2 + (1 + 2) + 2 + (1 + 2) = 12

9. (a) The energy-wave vector dispersion relation for a one dimensional crystal of lattice constant 'a' is given by $E(\kappa) = E_0 + 3\alpha\kappa^2 - 5\beta\kappa^4$, where E_0, α, β are positive constants. Find the expression for the group velocity of the electron within the crystal as a function of κ. For what value of κ the velocity is maximum?

(b) State and explain Bloch's theorem in one dimension.

(c) An electron is moving in one dimension periodic lattice with lattice constant 'a' with potential V(x) = V(x + a). If 'H' be the Hamiltonian of the electron and $\widehat{T_a}$ be the lattice translational operator, then
(i) Show that H is periodic function of x with periodicity 'a'.
(ii) Show that, if $\psi(x)$ is an eigen function of H with eigen vale E then "$\widehat{T_a}\psi(x)$" is also an eigen function of H with the same eigen value.

(2 + 2) + (2 + 2) + (2 + 2) = 12