B.TECH / IT /5TH SEM/ INFO 3133/2017
DISCRETE MATHEMATICS
(INFO 3133)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Let be the proposition 'He is intelligent' and be the proposition 'He is tall'. Then $(\sim\mathbf{q}\wedge\sim\mathbf{p})$ states that:
(a) He is either intelligent or tall
(b) He is neither tall nor intelligent
(c) He is intelligent but not tall
(d) He is intelligent and tall.

(ii) The proposition $\mathbf{p}\wedge(\mathbf{q}\wedge\sim\mathbf{q})$ is
(a) a contradiction
(b) a tautology
(c) neither a contradiction nor a tautology
(d) both a contradiction and a tautology.

(iii) The letters of APPLE can be arranged in how many ways?
(a) 30 (b) 40 (c) 50 (d) 60.

(iv) A connected planar graph with 5 edges determines 3 regions. The number of vertices of this graph is
(a) 3 (b) 4 (c) 5 (d) 6.

(v) If a is prime to b, then $(a + b)$ is prime to
(a) ab (b) $(a+b)^2$ (c) $(a+b)^3$ (d) $(a+b)^{10}$.

(vi) In the poset ({2, 4, 5, 10, 12, 20, 25},|) ('|' stands for divisibility) one of the maximal elements is
(a) 5 (b) 10 (c) 4 (d) 12.

(vii) Total number of non-negative integer valued solutions to the equation $x + y + z = 8$, $x, y, z \geq 0$ is
(a) 35 (b) 45 (c) 55 (d) 65.

(viii) The generating function for the sequence {2, 0, 2, 0, 2, 0……..} is
(a) $\frac{2}{1-2x}+\frac{1}{1-x}$ (b) $\frac{1}{1-x}+\frac{1}{1+x}$
(c) $\frac{1}{1-x}-\frac{1}{1+x}$ (d) $\frac{2}{1-2x}-\frac{1}{1-x}$.

(ix) The chromatic number of a graph containing an odd circuit is
(a) 4 (b) ≥4 (c) 3 (d) ≥3.

(x) A solution of the congruence is
(a) 3 (b) 2 (c) 4 (d) 1.

Group - B

2. (a) Construct the truth table for:
$$\big((\mathbf{p}\to\mathbf{q})\wedge(\mathbf{q}\to\mathbf{r})\big)\to(\mathbf{p}\to\mathbf{r}).$$

(b) Without constructing a truth table, prove that
$(\mathbf{p}\vee\mathbf{q})\wedge(\mathbf{p}\to\mathbf{r})\wedge(\mathbf{q}\to\mathbf{r})\wedge\sim\mathbf{r}\equiv\mathbf{F}$ (contradiction).

6 + 6 = 12

3. (a) Find the CNF (Conjunctive Normal Form) of the following statement
$\{\mathbf{q}\vee(\mathbf{p}\wedge\mathbf{r})\wedge\sim(\mathbf{p}\vee\mathbf{r})\wedge\mathbf{q}\}$
Show your work in detail.

(b) Prove by mathematical induction that the following proposition is true: $\mathbf{P(n)}$: "$\mathbf{n!>2^n\ \forall n\geq 4, n\in Z^+}$" where Z^+ denotes the set of all positive integers (stating clearly and in detail the inductive base, inductive hypothesis and inductive step.)

6 + 6 = 12

Group - C

4. (a) Determine whether the relation on the set of all integers is reflexive, symmetric, antisymmetric and transitive where $(x, y)\in R$ iff
(i) x≡ y (mod 7)
(ii) x is a multiple of y

(b) Let L be a distributive lattice. Show that if $\mathbf{a}\wedge\mathbf{x}=\mathbf{a}\wedge\mathbf{y}$ and $\mathbf{a}\vee\mathbf{x}=\mathbf{a}\vee\mathbf{y}$ for some a ∈ L, then x = y.

6 + 6 = 12

5.(a) Calculate gcd(567, 315) and express gcd(567, 315) as 567u + 315v where u , v are integers.

(b) Find the remainder when 1! + 2! + 3! +⋯......+50! is divided by 15.

(c) Prove that if $ax \equiv ay \pmod m$ and a is prime to m , then $x \equiv y \pmod m$

6 + 3 + 3 = 12

Group - D

6. (a) How many integers between 1 to 300 (both inclusive) are divisible by:
(i) at least one of 3, 5, 7
(ii) by 3 and 5 but not by 7

(b) (i) Find the number of integer solutions of the equation
$x_1 + x_2 + x_3 + x_4 = 21$ where $x_1 \geq 8$ & $x_2, x_3, x_4 \geq 0$
(ii)Find the number of integer solutions of the equation
$x_1 + x_2 + x_3 + x_4 = 21$ with $0 \leq x_1 < 8$ $x_2, x_3, x_4 \geq 0$.

(3 + 3) + (3 + 3) = 12

7. (a) Apply generating function technique to solve the following recurrence relation:

$$a_{n+2} - 7a_{n+1} + 10a_n = 0 \text{ for } n \geq 0,\ a_0 = 3, a_1 = 36.$$

(b) Solve the following recurrence relation:

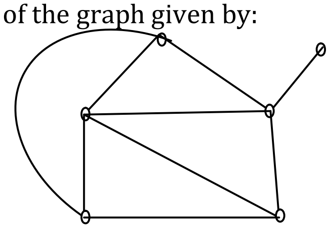
$$a_n - 4a_{n-1} + 3a_{n-2} = 2^n \text{ for } n \geq 2,\ a_1 = 1, a_2 = 11$$

6 + 6 = 12

Group - E

8. (a) Let G be a simple planar graph with n vertices and e edges, where $n \geq 3$. Then prove that $e \leq 3n - 6$.

(b) Draw the dual of the graph given by:



(c) The vertices of a connected graph has following degree sequence {2, 2, 2, 3 ,3, 3, 4, 4, 5}. If the graph is planar, how many faces will it have?

6 + 4 + 2 = 12

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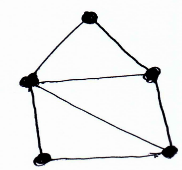
9. (a) Using Decomposition theorem, determine the chromatic polynomial of the following graph.



(b) Applicants M_1, M_2, M_3, M_4 apply for five posts P_1, P_2, P_3, P_4, P_5. The applications are done as follows :
$M_1 \to \{P_1, P_2\}$, $M_2 \to \{P_1, P_3, P_5\}$, $M_3 \to \{P_1, P_2, P_3, P_5\}$, $M_4 \to \{P_3, P_4\}$.
Using graph theory find
(i) whether there is any perfect matching of the set of applicants into the set of posts. If yes, find the matching.
(ii) whether every applicant can be offered a single post.

(c) Find the chromatic number of the wheel graph W_6.

6 + (3 + 2) + 1 = 12