(v) Worst case time complexity for inserting an element in a sorted array so that it stays sorted is
(a) O(1) (b) O(n) (c) $O(n^2)$ (d) None of these.

(vi) An algorithm is made up of 2 modules M1 & M2. If order of M1 is f(n), M2 is g(n), then the order of the algorithm is
(a) max(f(n), g(n)) (b) min(f(n), g(n))
(c) f(n) + g(n) (d) f(n) * g(n)

(vii) The time complexity of Counting sort with number of elements **n** and maximum element **m** is
(a) O(m logm) (b) O(n log m)
(c) O(n + m) (d) O(n – m)

(viii) Solution of the recurrence $T(n) = 2T(n/4) + \sqrt{n}$, T(1) = 1 for n>=0, n is of the form 4^k, k is an integer, is
(a) log n (b) $\sqrt{n}$ log n (c) n (d) n^2.

(ix) In a flow network, there exists a cut of value 11. Which of the following can be concluded?
(a) The minimum flow in the network is 11.
(b) The maximum flow in the network ≤ 11.
(c) The maximum flow in the network ≥ 11.
(d) None of these.

(x) Which one is true of the following:
(a) All NP-Hard problems are NP-Complete
(b) All NP-Complete problems are NP-Hard
(c) Some NP-Complete problems are NP-Hard
(d) None of these.

Group - B

2\. (a) Let's assume that a polynomial of degree m is represented as $t(n) = \sum_{i=0}^{m} a_i n^i$. Prove that $t(n) = O(n^m)$

(b) Prove that g(n) = Ω(f(n)), iff f(n) = O(g(n)), where all the symbols have their usual meaning w.r.t asymptotic complexity.

(c) Given a real number X and a sorted array S of n numbers, design an algorithm that, determines whether or not there exist two elements in S whose sum is exactly X. Justify that the time complexity of your algorithm is O(n).

4 + 6 + 2 = 12

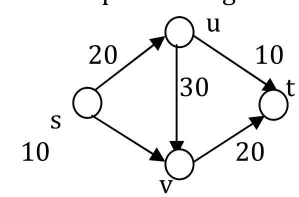
Group - E

8\. (a) Define P, NP, NP-Complete and NP-Hard problems. Show that VERTEX-COVER problem is NP-Complete.

(b) Give a polynomial-time 2-approximation algorithm for vertex cover problem and also prove that the algorithm indeed achieves that factor.

(2+4) + 6 = 12

9\. (a) Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represents capacity of the respective edge.



(b) Define the following terms in the context of optimization problems -
(i) Polynomial-time approximation scheme
(ii) Fully polynomial-time approximation scheme (FPTAS)

8 + (2 + 2) = 12

2016

DESIGN AND ANALYSIS OF ALGORITHMS
(CSEN 2201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following: **10 × 1 = 10**

(i) The number of edges in a DFS forest of a graph with 100 vertices, having 19 connected components is ________
(a) 19 (b) 81 (c) 80 (d) None of the above.

(ii) In the KMP algorithm for pattern matching, the suffix function $\sigma(x)$ is the _____est ______ of the pattern P that is also a ________ of x.
(a) large, prefix, suffix (b) small, prefix, suffix
(c) large, suffix, prefix (d) large, suffix, prefix

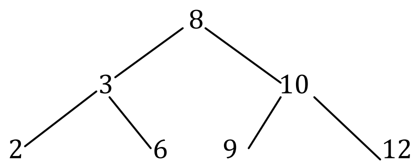
(iii) For simultaneously finding the minimum and maximum of n elements, the number of comparisons required is at most
(a) $3\lfloor n/2 \rfloor$ (b) 2n (c) 2 (n -1) (d) n log n.

(iv) A student proved that the longest path problem is NP-complete by reducing it to another already known NP-complete problem named set-cover problem. His teacher said the proof is not correct and did not give him any marks. Which of the following is true -
(a) The teacher does not understand NP-completeness as it is a difficult chapter.
(b) The student got a wrong answer about the hardness of longest path problem.
(c) The method of proof given by the student was wrong.
(d) The teacher is biased and hence did not give marks to the student.

3. (a) Show that the 2nd smallest of **n** elements can be found with $\mathbf{n} + \lceil \mathbf{lgn} \rceil$ - 2 comparisons in the worst case.

(b) State the asymptotic average case complexities of bubblesort, heapsort, mergesort and binary search?
A binary tree can be represented as an array where the left child of element i is at index
2*i +1 and right child is at 2*i + 2. Write a procedure (pseudo code) that will use binary search to locate an item in this array. As for example the following table represents the following tree.

Array Index	0	1	2	3	4	5	6
Contents	8	3	10	2	6	9	12



5 + 2 + 5 = 12

Group - C

4. (a) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is <30, 35, 15, 5, 10, 20, 25> using dynamic programming method.

(b) Let's suppose that you are given a set of N jobs. Each job i is associated with an integer deadline d_i>=0 and a profit p_i>0. For any job i, the profit p_i is earned iff the job is completed by its deadline, and to complete a job, one has to process the job on a machine for 1 unit of time. Only one machine is available for processing the jobs. The feasible solution for this problem is a subset of jobs such that each one can be completed by its deadline. Optimal solution is a feasible one with maximum profit value.
Now Give a greedy algorithm for the above problem so that optimal solution can be obtained. Also analyze your algorithm in worst case.

7 + 5 = 12

5. (a) State the longest common subsequence (LCS) problem.
Give a simple plan/algorithm to calculate the longest monotonically increasing sequence of n distinct integers in $O(n^2)$ time.

(b) Give a small example showing that Minimum Spanning Tree (MST) of a connected undirected graph may not be unique.

B.TECH/CSE/4TH SEM /CSEN 2201/2016

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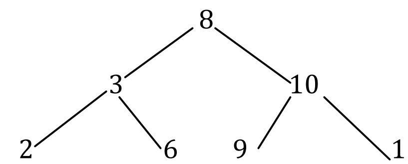

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CSEN 2201 3

What is the main difference between the Kruskal's and Prim's algorithm?

State the time complexity of the Prim's algorithm for both cases where we use (i) an array, (ii) a heap to implement the priority queue.

(2+4) + (2+2+2) = 12

Group - D

6. (a) A sequence of n operations is performed on a data structure. The cost of ith operation is $C(i) = i^2$, if i is an exact power of 3

$= 3$, otherwise.

Calculate the exact expression for cost for n successive operations. Use Aggregate Analysis to determine the amortized cost per operation.

(b) Give the asymptotic complexities for string matching using finite automata and KMP algorithm. Define the terms used in the expression.

(c) Give the pseudo code for identifying the strongly connected components in a graph.

You can use DFS(G) as a procedure that is already available to you. DFS(G) stands for Depth First Search on graph G.

6 + (1+1) + 4 = 12

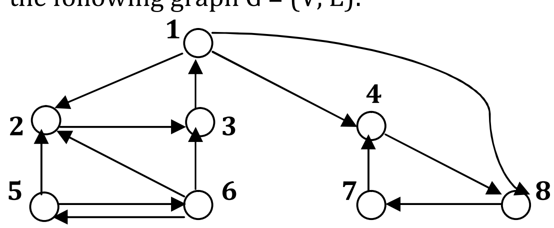
7. (a) Given the following:

String S: bacbababababacaca

Pattern P: ababaca

Show how Knuth-Morris-Pratt algorithm works to solve the string matching problem for the above case

(b) Consider the following graph G = (V, E):



Find the strongly connected components of the above graph. Also justify that the strategy that you have applied here produces the strongly connected components in O (V + E) time.

6 + (4+2) = 12

B.TECH/CSE/4TH SEM /CSEN 2201/2016

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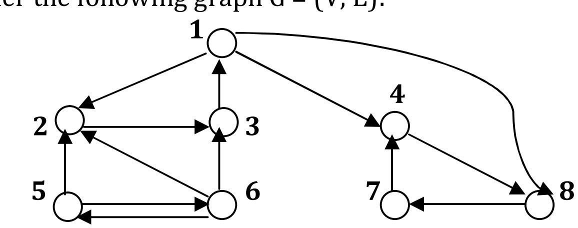

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