## B.TECH/CSE/IT/7TH SEM/ MATH 4181/2017

## OPERATIONS RESEARCH AND OPTIMIZATION TECHNIQUES (MATH 4181)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

### Group – A
### (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If at least one of the basic variable is zero, then a basic feasible solution of an L.P.P. is
(a) degenerate (b) non-degenerate
(c) infeasible (d) unbounded.

(ii) The dual of the dual of an LPP is
(a) dual (b) primal (c) nonlinear (d) a different LPP.

(iii) In a transportation problem of size m × n a feasible solution is called a basic solution if the number of non-negative allocation is equal to
(a) $m - n + 1$ (b) $m - n - 1$ (c) $m + n - 1$ (d) none of these.

(iv) In the optimal simplex table if any of the non basic variables attains a zero net evaluation then the LPP has
(a) unbounded solution (b) no feasible solution
(c) unique optimal solution (d) alternative solution.

(v) To standardize an L.P.P. with "≤" type constraints, which variables are introduced
(a) slack (b) surplus (c) artificial (d) unrestricted.

(vi) Which of the following elimination methods is most efficient to find the local optima of an unimodal function of one variable?
(a) Golden section method (b) Fibonacci Method
(c) Interval Halving (d) Dichotomous Search.





(vii) The point of intersection of pure strategies in a game is called
(a) value of the game (b) saddle point
(c) mixed strategy (d) optimal strategy.

(viii) The function $f(x,y) = xy^2(2 - x - y)$ has, at point (0, 2)
(a) a local maximum point (b) a local minimum point
(c) a saddle point (d) none of these.

(ix) The Hessian matrix of function f(x, y) is given by
$$Hf(x,y) = \begin{pmatrix} x^2+2 & -1 \\ -1 & 1 \end{pmatrix}.$$
If $f(x,y)$ had a stationary point then this point would be
(a) a global maximum point (b) a global minimum point
(c) a saddle point (d) none of these.

(x) Given the optimization problem
$$\min f(x,y)$$
$$\text{subject to } 3x - 6y = 9$$
If $(x,y,\lambda) = (1,-1,3)$ is a stationary point of the associated Lagrange function, it can be assured that $(1,-1)$ is a global minimum of the problem when the function $f(x,y)$ is
(a) onvex (b) non-convex
(c) concave (d) neither convex nor concave

### Group – B

2. (a) Solve the following L.P.P. graphically
$$\text{Minimize } z = 20x_1 + 10x_2$$
subject to
$$x_1 + 2x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 60$$
$$x_1,\ x_2 \ge 0$$

(b) Solve the following L.P.P. by Simplex method
$$\text{Maximize } z = x_1 + x_2 + 3x_3$$
subject to
$$3x_1 + 2x_2 + x_3 \le 3$$
$$2x_1 + x_2 + 2x_3 \le 2$$
$$x_1,\ x_2,\ x_3 \ge 0$$

**4 + 8 = 12**

3. (a) Use the 'Big-M' method to solve the following linear programming problem:

$$\text{Maximize } z = 4x_1 + 5x_2$$
Subject to
$$x_1 + x_2 \le 4$$
$$2x_1 + 3x_2 \ge 12$$
$$4x_1 + 3x_2 \ge 18$$
$$x_1, x_2 \ge 0$$

(b) Write the dual of the following LPP:

$$\text{Maximize } z = 2x_1 + 3x_2 - 4x$$
Subject to
$$3x_1 + x_2 + x_3 \le 2$$
$$-4x_1 + 3x_3 \ge 4$$
$$x_1 - 5x_2 + x_3 = 5$$
$x_1, x_2 \ge 0$ and $x_3$ is unrestricted in sign

**8 + 4 = 12**

## Group – C

4. (a) Obtain an initial basic feasible solution and total cost of transportation to the following Transportation problem using North-West corner rule. Is that initial basic feasible solution non-degenerate?

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $O_1$ | 4 | 6 | 8 | 8 | 40 |
| $O_2$ | 6 | 8 | 6 | 7 | 60 |
| $O_3$ | 5 | 7 | 6 | 8 | 50 |
| Demand | 20 | 30 | 50 | 50 | |

(b) Find the optimal assignment and minimum cost for the assignment with the following cost matrix

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 6 | 5 | 8 | 11 | 16 |
| B | 1 | 13 | 16 | 1 | 10 |
| C | 16 | 11 | 8 | 8 | 8 |
| D | 9 | 14 | 12 | 10 | 16 |
| E | 10 | 13 | 11 | 8 | 16 |

**5 + 7 = 12**

5. (a) Use dominance to reduce the following game problem to a 2 × 2 game and hence find the optimal strategies and the value of the game





**Player B**

| **Player A** | | | |
|---|---|---|---|
| | 1 | 7 | 2 |
| | 0 | 2 | 7 |
| | 5 | 1 | 6 |

(b) Solve graphically the game whose payoff matrix is given by

**Player B**

| **Player A** | | | | |
|---|---|---|---|---|
| | 1 | 3 | 0 | 2 |
| | 3 | 0 | 1 | -1 |

**6 + 6 = 12**

## Group – D

6. (a) Find the point $(x_1, x_2, x_3)$ at which the following function
$$f(x_1, x_2, x_3) = -x_1^2 - x_2^2 - x_3^2 + x_1x_2 + x_1 + 2x_3$$
attains a local optima.

(b) Solve the following non-linear programming problem using Lagrange multiplier method:

$$\text{Minimize } f(x_1, x_2, x_3) = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$
Subject to the constraints
$$x_1 + x_2 + x_3 = 15$$
$$2x_1 - x_2 + 2x_3 = 20$$
$$x_1, x_2, x_3 \ge 0$$

**8 + 4 = 12**

7. (a) Maximize $f(x_1, x_2) = 10x_1 - x_1^2 + 10x_2 - x_2^2$
Subject to the constraints
$$x_1 + x_2 \le 8$$
$$-x_1 + x_2 \le 5$$
$$x_1, x_2 \ge 0$$
by applying Kuhn-Tucker conditions.

(b) Show that the function $f(x_1, x_2) = x_1x_2 - x_1^2 - x_2^2$ is concave over $\mathbb{R}^2$.

**10 + 2 = 12**

## Group – E

8. Write the Dichotomous Search algorithm and use the algorithm
$f(x) = -x^2 - 2x$ over $[-3, 6]$
assuming the optimal tolerance to be less than 0.2.

**12**

9. Write the Golden Section Search Algorithm for unimodal functions of one variable and using the algorithm maximize $f(x) = -x^2 - 2x$ over $-3 \le x \le 6$ with a tolerance to be less than 0.2.

**12**