# GRAPH ALGORITHMS
# (CSEN 4163)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
## (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following problem is polynomial-time solvable?
(a) Longest Path problem
(b) Finding a Hamiltonian circuit in a given graph
(c) Finding an Eulerian circuit in a given graph.
(d) None of the above.

(ii) In a flow network, there exists a cut of value 11. Which of the following can be concluded?
(a) The minimum flow in the network is 11.
(b) The maximum flow in the network ≤ 11.
(c) The maximum flow in the network ≥ 11.
(d) None of these.

(iii) Solution of TSP is the
(a) Longest Hamiltonian cycle (b) Longest Eulerian cycle
(c) Shortest Hamiltonian cycle (d) Shortest Eulerian cycle.

(iv) Stable set is
(a) a set of edges with no common vertex
(b) a set of vertices which are not adjacent
(c) a set of vertices with even degree
(d) none of the above.

(v) Which of the following can be a valid degree sequence for a simple undirected graph with $|V| = 6$?
(a) 5, 5, 4, 3, 2, 1 (b) 5, 4, 4, 3, 2, 1
(c) 6, 4, 4, 3, 2, 1 (d) none of these.





(vi) If none of the regions in a plane graph in bounded by an odd number of edges, which of the following is correct -
(a) $2r \le e$ (b) $3r \le e$
(c) $4r = e$ (d) None of the above.

(vii) The chromatic polynomial $P(G, t)$ where $G = K_5$ is -
(a) $t^5$ (b) $t(t-1)^4$
(c) $t(t-1)^2(t-2)^2$ (d) none of the mentioned.

(viii) Perfect graphs are the same as
(a) Peterson graphs (b) Berge graphs
(c) Cayley graphs (d) Lattice graphs.

(ix) The cardinality of a perfect matching in a hypercube of order 5 is equal to
(a) 8 (b) 16 (c) 24 (d) 32.

(x) Which of the statements is/are true?
(I) A plane graph has a unique dual graph
(II) A planar graph can have multiple dual graphs
(a) only I
(b) only II
(c) both (I) and (II)
(d) does not make sense as plane is same as planar.

## Group – B

2. (a) Assuming that the routine for doing a DFS in a directed graph is available to you, give the pseudo code for finding the strongly connected components in a directed graph.

(b) Let C and C' be two distinct strongly connected components in a directed graph $G = (V, E)$, let $u, v \in C$, and $u', v' \in C'$, and suppose that G contains a path from u to u'. Then prove that G cannot contain a path from v' to v.

(c) How can the number of strongly connected components of a directed graph G change if a new edge is added? Justify your answer.

(d) What are the conditions required for having
(i) an Eulerian Tour in an undirected graph with no Eulerian cycle possible.
(ii) an Eulerian cycle in a directed graph.

**3 + 4 + 3 + 2 = 12**

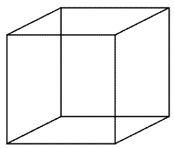
3. (a) If you know beforehand that the graphs on which your algorithm is running has $|E| = O(|V|^2)$. Which graph representation technique out of the two most popular ones will you use and why? Will you change your decision if $|E| = O(|V|.\log|V|)$? Justify.

(b) State the Travelling Salesman Problem (TSP). What is the triangular TSP? Give a natural example of the triangular TSP? Give a constant approximation algorithm for the triangular TSP.

**4 + (2 + 1 + 1 + 4) = 12**

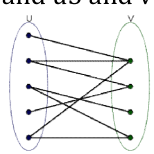
## Group – C

4. (a) (i) Define maximal matching and perfect matching.
(ii) Write all the different perfect matchings in a cube (diagram given below), with the vertices being labelled as a, b, c, d, e, f, g, and h.
Hint: The number of such matchings is a perfect square.



(iii) Can you give an example of a matching that is maximal but not perfect in the above graph?
(iv) The number of such matchings in the question above is 8. Can you list all of them?

(b) Can you suitably enhance the following undirected un-weighted bipartite graph so that you can then run the max-flow algorithm on it to determine the size of maximum bipartite matching? Please name the vertices as u1, u2, u3, u4 and u5 and v1, v2, v3 and v4.



**(2 + 3 + 1 + 3) + 3 = 12**

5. (a) State with an example, what an unstable pair is with respect to the stable matching algorithm.

(b) Give the pseudo-code for Gale-Shapeley algorithm for finding a set of stable matching pairs. Assume that there are n men, n women and each of them has provided his/her full set of preferences.

(c) Prove that the Gale-Shapeley algorithm in the above setting terminates after at most $n^2$ iterations of the while loop.

(d) State the asymptotic time complexity of Hopcroft-Karp algorithm for finding maximum matching.

**2 + 6 + 1 + 3 = 12**



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## Group – D

6. (a) If a graph contains an odd cycle, what is the lower bound on the chromatic number of the graph? Justify.

(b) A graph that contains no odd cycles is n-colorable. What is the smallest value of n? Justify your answer. You may use this famous result – 'A graph is bipartite if and only if all its cycles are even'.

(c) What is a perfect graph?

(d) A *hole* is a chordless cycle of length at least four; an *antihole* is the complement of such a cycle. Show that -
(i) An even hole is always perfect.
(ii) No odd hole is perfect
(iii) The clique number of an odd antihole with 2k + 1 vertices is k.
(iv) The chromatic number of an odd antihole with 2k + 1 vertices is k +1.

**2 + 2 + 2 + (1 + 1 + 2 + 2) = 12**

7. (a) State Kuratowski's Theorem.

(b) Show that for any planar graph $3r/2 \le e \le 3n - 6$, where n, e and r are the number of vertices, edges and regions respectively. Note that the unbounded region is also counted within r.

(c) State the Perfect Graph Theorem.

(d) Prove that for any graph G = (V, E) $|I| + |C| = |V|$, where I is a maximum independent set and C is a minimum vertex cover of the graph.

**2 + 6 + 1 + 3 = 12**

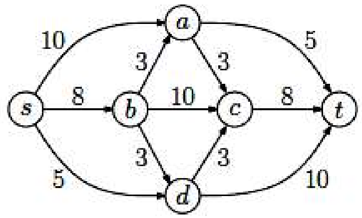
## Group – E

8. (a) Define Interval graph and Perfect elimination ordering (P.E.O) with a proper example. Prove that there is an efficient algorithm to solve graph colouring problem on graphs with P.E.O.

(b) Prove the following lemma:
A random graph $G_{n,p}$, given that its number of edges is m, is equally likely to be one of the ((n choose 2) choose m) graphs that have m edges.

**(2 + 2 + 2) + 6 = 12**

9. (a) Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denote the source & the destination and the weights associated with every edge represents capacity of the respective edge.




(b) Give two limitations of the Ford-Fulkerson algorithm, for which one may prefer Edmonds-Karp algorithm.

(c) Suppose you have run a max-flow algorithm for some time on a graph G and there exists no more augmenting path, i.e. the flow has reached saturation. You may assume that the residual graph is available to you. Now, you would like to determine the cut, i.e., you want to determine the vertices in source partition and sink partition. Give a simple algorithm / pseudo-code that you can run to determine the cut.
Hint: This is not at all difficult.

**4 + 2 + 6 = 12**



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