PROBABILITY AND NUMERICAL METHODS
(MATH 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In Newton's backward interpolation formula, the value of $u = \frac{x - x_n}{h}$ lies belween

(a) 1 and 2 (b) - 1 and 0

(c) 0 and 1 (d) - 1 and 1

(ii) The degree of the approximating polynomial corresponding to Simpson's $\frac{1}{3}$ rule is

(a) 2 (b) 1 (c) 3 (d) 4

(iii) Gauss Seidel method for the solution of a system of linear simultaneous equations converges if

(a) $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$ (b) $|a_{ii}| < \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$ (c) $\frac{|a_{ii}|}{|a_{nn}|} = 1$ (d) none of these

(iv) Which of the following does not always guarantee convergence?

(a) secant method (b) bisection method

(c) regular falsi method (d) Newton- Raphson method

(v) The iterative method to solve a system of equations is

(a) Gauss elimination (b) LU decomposition

(c) Gauss Jordan (d) Gauss Seidel

(vi) When the variance of a random variable X is $\frac{2}{3}$, then $V(3X+5)$ is:

(a) 6 (b) 7 (c) 5 (d) 8

(vii) A fair coin is tossed successively three times. The probability of getting exactly one head is:

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) *none*

(viii) The p.d.f of a random variable X is

$$f(x) = \begin{cases} k(2x-1) & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

The value of the constant k is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(ix) The mean and standard deviation of a Binomial distribution are 24 and 16 respectively. Then the parameters n and p are

(a) $72, \frac{1}{3}$ (b) $72, \frac{2}{3}$

(c) $48, \frac{1}{2}$ (d) $48, \frac{2}{3}$

(x) $P_1 = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1/2 & 1/3 \\ 1/2 & 2/3 \end{bmatrix}$, $P_3 = \begin{bmatrix} 1/3 & 2/3 \\ 1/2 & 1/2 \end{bmatrix}$

Out of these three matrices which ones can possibly be transition probability matrix:

(a) all (b) P_1 and P_2 (c) P_2 and P_3 (d) P_3 and P_1

Group – B

2. (a) Find the cube root of 10 correct upto three decimal places using a suitable iterative method.

(b) Solve the following system of equations by LU decomposition method

$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$
$$3x + y + 2z = 16$$

5 + 7 = 12

(b) (i) State Chapman - Kolmogorov equation.

(ii) $X_0, X_1, X_2, \ldots\ldots$ forms a markov chain where each X_i can only assume numerical values 0, 1 or 2. The transition probability matrix of the markov chain is :

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 1/3 & 0 & 2/3 \\ 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{matrix}$$

Find $P(X_8 = 2 / X_6 = 0)$ and $P(X_5 = 0 / X_3 = 2)$.

(1 + 5) + (2 + 2 + 2) = 12

3. (a) Applying suitable interpolation formula, find $\sqrt{2}$ correct up to 4 significant figures from the following table:

x	1.9	2.1	2.3	2.5	2.7
$\sqrt{x}$	1.3784	1.4491	1.5166	1.5811	1.6432

(b) Find the value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2 dx}{1+x^3}$ using Simpson's $1/3$ rule with $h = 0.25$ correct upto 4 significant figures.

6 + 6 = 12

Group – C

4. (a) Solve by Euler's modified method, the differential equation: $\frac{dy}{dx} = x^2 + y,\ \ y(0) = 1$ for $x = 0.02$ taking step length $h = 0.01$ (correct upto 3 decimal places).

(b) Find the values of $y(0.2)$ and y(0.4) using the Runge - Kutta method of the fourth order, given that $\frac{dy}{dx} = xy + y^2,\ y(0) = 1$

6 + 6 = 12

5. (a) Two players A and B alternately throw a pair of dice. The total of the two dice is taken for each of them. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A starts the game then find his probability of winning.

(b) A random variable X has the probability density function

$f(x) = \frac{a}{x^2 + 1}$ for $-\infty < x < \infty$,

where a is a constant. Find the probability that X^2 lies between $\frac{1}{3}$ and 1.

(3 + 3) + 6 = 12

Group – D

6. (a) The inner diameter of the tubes produced by a machine follows Normal distribution with mean 0.502 inch and standard deviation 0.005 inch. The purpose for which these tubes are intended allows a maximum tolerance for diameter to be in the range of 0.496 to 0.508 inch. (i.e., otherwise the tubes are considered defective). What percentage of the tubes produced by the machine is defective? (Area under the standard normal curve between $z = 0$ and $z = 1.2$ is 0.3849)

(b) If the random variable X follows Poisson distribution with parameter λ ($\lambda > 0$) then derive the expression for $E(X)$ and $V(X)$.

(3 + 3) + (3 + 3) = 12

7. (a) An incomplete frequency distribution is given as follows:

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	?	65	?	25	18

It is known that the total frequency is 229 and the median of the data is given to be 45. Determine the missing frequencies and also the mode.

(b) From the following data determine the regression equation of y on x:

x	5	7	9	11	13	15
y	1.70	2.40	2.80	3.40	3.70	4.40

(4 + 2) + 6 = 12

Group – E

8. (a) The random variable X and Y have joint p.m.f. $f(x, y) = \dfrac{2x + y}{27}$ for $x, y = 0,\ 1, 2.$

Find $E(X)$ and $E(Y)$. Are X and Y independent?

(b) The random variables X and Y have joint probability density function:

$$f(x, y) = \begin{cases} e^{-(x+y)} & \text{for } x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(1 < X + Y < 2)$.

6 + 6 = 12

9. (a) A box contains 4 balls. Every ball is either white or red. Two balls are randomly picked up and are replaced by two other balls of complementary color (which means if both the drawn balls are red then they are replaced by two white balls, if both drawn balls are white then they are replaced with two red and if one white and one red are drawn then they are replaced with one red and one white). Let X_n be the number of white balls in the box after this process is carried out n times. Find out the state space and the transition probability matrix of this Markov chain.