B.Tech/CSE/3rd Sem/CSEN-2102/2015

2015

DISCRETE MATHEMATICS

(CSEN 2102)

Time Alloted : 3 Hours *Full Marks : 70*

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : **[10×1=10]**

i) $(p \to q) \leftrightarrow (\sim p \lor q)$ is a

(a) tautology (b) contradiction
(c) contingency (d) none of these

ii) From the premise $\sim q$ and $p \to q$ one can conclude

(a) p (b) q
(c) $\sim p$ (d) $\sim q$

iii) In how many ways can 8 ladies and 8 gentlemen sit around a circular table with no two ladies being together?

(a) 7! 8! (b) 7^8
(c) 8! 8! (d) 7!

iv) Solution of $a_k = 3a_{k-1}$, $k \geq 1$ with $a_0 = 2$ is

(a) 3^n (b) 3n (c) $(3!)^n$ (d) 2.3^n

v) The number of ways a complete graph having n vertices can be coloured with n – 1 colours is

(a) n – 1 (b) 1
(c) (n – 1)! (d) 0

vi) A graph has n vertices. The maximum possible number of components it can have is

(a) 2 (b) n – 1 (c) 1 (d) n

vii) $x^4 - 3x + 2x^2 + x - 1$ is

(a) the chromatic polynomial of a complete graph
(b) the chromatic polynomial of a tree
(c) not the chromatic polynomial of any graph
(d) the chromatic polymonial of a bipartite graph

viii) Let G be a graph and let G* be its dual. The number of edges of G* is equal to

(a) the number of vertices of G
(b) the number of edges of G
(c) twice the number of vertices of G
(d) the number of regions of G

ix) The value of ${}^{50}C_0\,{}^{50}C_1 + {}^{50}C_1\,{}^{50}C_2 + \ldots\ldots + {}^{50}C_{49}\,{}^{50}C_{50}$ is

(a) ${}^{100}C_{50}$ (b) ${}^{100}C_{51}$
(c) ${}^{50}C_{25}$ (d) ${}^{50}C_{25}\,{}^{50}C_{25}$

x) Let C(x) be the statement "x is a comedian" and F(x) be the statement "x is funny". Consider the English Statement "x is funny and x is a comedian". The translation of this into a logical statement is

(a) $\exists x(C(x) \to F(x))$ (b) $\forall x(C(x) \to F(x))$
(c) $\exists x(C(x) \wedge F(x))$ (d) $\forall x(C(x) \wedge F(x))$

GROUP - B

2. (a) Define converse, inverse and contra-positive of an implication. Prove that the following pair of propositon is equivalent.

$$\sim (p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \text{ and } p \rightarrow q$$

(b) Check the validity of the following argument :

"If I get the job and work hard, then I will be promoted. If I get promoted, then I will be happy. I am not happy. Therefore, either I will not get the job or I will not work hard". **(2+4)+6 = 12**

3. (a) Prove the implication:

$$\forall x(P(x) \rightarrow Q(x)), \forall x(R(x) \rightarrow \sim Q(x)) \Rightarrow \forall x(R(x) \rightarrow \sim P(x))$$

(b) Find the conjunctive normal form of the following statement :

$$(p \wedge \sim (q \wedge r)) \vee (p \rightarrow q)$$

(c) Construct the truth table of the statement :

$$(\sim p \leftrightarrow \sim q) \leftrightarrow (q \leftrightarrow r)$$

6+3+3 = 12

GROUP - C

4. (a) Show that among n+1 positive integers not exceeding 2n, there must be an integer that divides one of the other integers.

(b) In a group of 2092 people, 1232 know driving, 879 know swimming and 114 can play musical instrument. It is known that 103 know both driving and swimming, 23 can swim and also play a musical instrument and 14 can drive and play a musical instrument. How many people can drive, swim and play musical instrument?

(c) Prove that $f_n | f_{2n}$ (i.e., f_n divides f_{2n}), where f_n is the n-th Fibonacci number ($f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$ for n>2). **4+3+5 = 12**

5. (a) Solve using generating functions the recurrence relation

$$a_n - 7a_{n-1} + 10a_{n-2} = 2, \text{ for all } n > 1, \text{ with } a_0 = 3, a_1 = 3$$

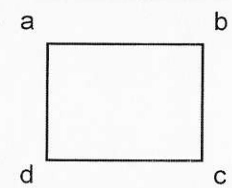
(b) Let n be a positive integer. Then prove that

$$\sum_{k=0}^{n} (-1)^k C(n,k) = 0$$

(c) What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$? Show your work. **6+3+3 = 12**

Group - D

6. (a) Briefly explain chromatic polynomial of a simple graph. Find the chromatic polynomial of the following graph.

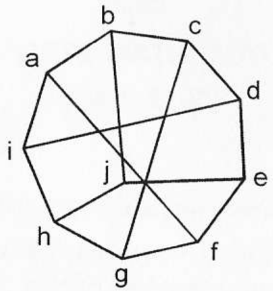


(b) Prove that every component of the symmetric difference of two matchings is a path or an even cycle.

(c) Take C_5 (cycle having 5 vertices) and K_3 (complete graph of 3 vertices) and join each vertex of C_5 to each vertex of K_3. Find the clique number and the chromatic number of the resulting graph. **5+3+4 = 12**

7. (a) List all subsets of edges that form maximal matchings in the wheel W_7.(Number the vertices on the perimeter 1, 2, 3, 4, 5,6 with the vertex in the centre being numbered 7). How many of these maximal matchings are perfect matching?

(b) What is an independent set of vertices of a simple graph? How it can be used in finding the chromatic number of a graph? Find the chromatic number of the following graph using this method.



5+(2+2+3) = 12

GROUP - E

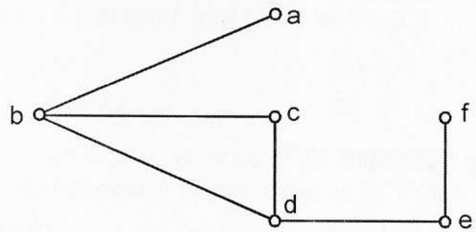
8. (a) Prove that an Eulerian graph cannot have a bridge.

(b) Prove by induction that a connected planar graph G with n vertices and e edges determines f = e – n + 2 regions.

5+7 = 12

9. (a) There are four processors p_1, p_2, p_3 and p_4 which are designated for six tasks t_1, t_2, t_3, t_4, t_5 and t_6 as follows: $p_1 \rightarrow \{t_1, t_2\}$, $p_2 \rightarrow \{t_1, t_3, t_4\}$, $p_3 \rightarrow \{t_3, t_4, t_5\}$, and $p_4 \rightarrow \{t_2, t_6\}$. Using Hall's Theorem find whether it is possible to develop a super processor which will serve exactly one task assigned to each of the four processors. Solve this problem graphically.

(b) Find the maximal and maximum matchings and matching number of the following graph :



Determine if there exists any perfect matching in the graph.

6+6 = 12