2015

MATHEMATICS 1

(MATH 1101)

Time Alloted : 3 Hours *Full Marks : 70*

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : **[10×1=10]**

i) The eigenvalues of the matrix A are a and b, then the eigenvalues of A^2 are

(a) ab, b^2 (b) a^2, b
(c) a^2, b^2 (d) a, b

ii) The maximum value of the determinant among all 2 × 2 real symmetric matrices with trace 14 is

(a) 48 (b) 49
(c) 41 (d) 14

iii) The value of $\begin{vmatrix} 1 & 1 & 1 \\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^{m+1}C_2 & {}^{m+2}C_2 & {}^{m+3}C_2 \end{vmatrix}$ is

(a) 1 (b) 0
(c) m (d) 2

iv) The series $\sum_{n=1}^{\infty} \mathrm{cosec}^n x,\ 0 < x < \frac{\pi}{2}$ is

(a) convergent (b) divergent
(c) oscillates finitely (d) oscillates infinitely

v) The sequence $\{u_n\}$ where $u_n = \frac{1+n}{n}$ is

(a) monotone increasing (b) monotone decreasing
(c) oscillatory (d) constant

vi) If y = log(1 + x), then $(y_n)_0$ is

(a) n! (b) $(-1)^n$ n!
(c) $(-1)^{n-1}$ (n − 1)! (d) $(-1)^n$ (n − 1)!

vii) If C is the circle $x^2 + y^2 = 1$, then $\int_C (xdx + ydy)$ is

(a) 1 (b) 0
(c) 2 (d) 3

viii) If $\phi = x^2y + 2xy + z^2$, Curl grad ϕ =

(a) 1 (b) 2
(c) 0 (d) 3

ix) If f(x, y) = 0, then $\frac{dy}{dx}$ is

(a) $\frac{f_x}{f_y}$ (b) $-\frac{f_x}{f_y}$

(c) $\frac{f_y}{f_x}$ (d) $-\frac{f_y}{f_x}$

x) If f(x,y) is a homogenous function of degree 5 then

$x\frac{\delta f}{\delta x}+y\frac{\delta f}{\delta y}=$

(a) 5f (x, y) (b) f (x, y)

(c) 0 (d) 5

GROUP - B

2. (a) Determine the values of a and b for which the system

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

has

(i) no solution

(ii) unique solution

(iii) infinite number of solutions

(b) Show that $\begin{vmatrix} a^2+\lambda & ab & ac & ad \\ ab & b^2+\lambda & bc & bd \\ ac & bc & c^2+\lambda & cd \\ ad & bd & cd & d^2+\lambda \end{vmatrix}$ is divisible by λ^3 and find the remaining factor.

B.Tech/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/1st Sem/MATH-1101/2015

(c) A. B, C are square matrices, each of order n, such that $AB = I_n$ and $CB = I_n$, then show that A = C.

(5)+(4)+(3) = 12

3. (a) Find the rank of the matrix $\begin{bmatrix} a & -1 & -1 \\ -1 & a & -1 \\ -1 & -1 & a \\ 1 & 1 & 1 \end{bmatrix}$ when

(i) $a \neq -1$ and (ii) $a = -1$.

(b) If λ is an eigen value of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is an eigen value of adj(A).

(c) Determine conditions under which the system of equations

$$x + 4y + 2z = 1$$
$$2x + 7y + 5z = 2b$$
$$4x + ay + 10z = 2b + 1$$

have (i) only one solution (ii) no solution (iii) many solutions.

3+3+6 = 12

GROUP - C

4. (a) If $f(x) = (x - a)^m (x - b)^n$ where m and n are positive integers, show that 'c' in Rolle's theorem divides the segment $a \leq x \leq b$ in the ratio m : n.

(b) In the Mean Value Theorem $f(h) = f(0) + hf'(\theta h)$, $0<\theta<1$, show that when $f(x) = \cos x$ then

$$\lim_{h\to 0+}\theta = \frac{1}{2}$$

(c) Find the nature of the infinite series given by $\sum_{n=1}^{\infty} r^{n-1}$ for all values of r.

4+4+4 = 12

5. (a) Examine the convergence of the following infinite series

$$\frac{1^2+2}{1^4}x+\frac{2^2+2}{2^4}x^2+\frac{3^2+2}{3^4}x^3+\ldots\ldots,\ x > 0.$$

(b) Using Mean Value Theorem, prove that

$$\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2} \text{ where } 0 < a < b < 2.$$

Hence deduce $\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$

(c) Find the Taylor's series expansion of the function $f(x) = a^x$ about $x = 0$, $a > 0$ with Lagrange's form of remainder after n terms.

5+5+2 = 12

Group - D

6. (a) Does the limit $Lt_{\substack{x\to 0\\ y\to 0}} \frac{x^2y^4}{(x^2+y^4)^2}$ exist?

(b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \sin 2u$

(c) If $x^2 + y^2 + u^2 - v^2 = 0$ and $uv - xy = 0$. Find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.

(3)+(5)+(4) = 12

7. (a) Find the extrema of the function

$$f(x,y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$$

(b) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = xf\left(\frac{y}{x}\right)$

(c) If $u = ax + by$, $v = bx - ay$, show that

$$\left(\frac{\partial u}{\partial x}\right)_y\cdot\left(\frac{\partial x}{\partial u}\right)_v\cdot\left(\frac{\partial y}{\partial v}\right)_x\cdot\left(\frac{\partial v}{\partial y}\right)_u = 1$$

5+3+4 = 12

GROUP - E

8. (a) Show that $\int_0^{\pi/2}\cos^{n-2}x\sin nx\,dx = \frac{1}{n-1}$, n being a positive integer greater than 2.

(b) Find constants a, b, c so that

$$V = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$$

is irrotational. Further show that V can be expressed as the gradient of a scalar function.

(6)+(6) = 12

9. (a) Verify the Stokes theorem for $\vec{F} = (x^2 + y^2)\,\hat{i} - 2xy\,\hat{j}$, taken round the rectangle bounded by the lines $x = -a$, $x = a$, $y = 0$, $y = b$.

(b) Show that $\int_0^{\pi/2} \cos^n x \cos nx \, dx = \frac{\pi}{2^{\pi+1}}$.

7+5 = 12
