(vii) In bisection method, the absolute error propagation after n[th] iteration (E_a^n) is given by ________, where Δx^0 is the difference between the initial guess values for upper and lower bound.

(a) $E_a^n = \frac{\Delta x^0}{2^{n-1}}$ (b) $E_a^n = \frac{\Delta x^0}{2^n}$ (c) $E_a^n = \frac{\Delta x^0}{2^{n-2}}$ (d) $E_a^n = \frac{\Delta x^0}{2}$

(viii) Heun's Method of ODE integration

(a) is similar to 2nd order Runge Kutta
(b) is similar to 1st order Runge Kutta
(c) is modified Euler method
(d) is a separate method from the methods said in (a), (b) and (c)

(ix) The truncation error for first order Taylor series approximation of $f(x) = x^2 - 3x$ is given by

(a) $h^2/2$ (b) $-2 - h + 2h^2$ (c) $1/2h$ (d) $-2 - h$

where 'h' is difference interval in 'x'.

(x) The Explicit scheme for solving the heat conduction equation is both convergent, stable and will not have oscillating solutions if

(a) $\Delta t \le \frac{\Delta x^2}{k}$ (b) $\Delta t \le \frac{\Delta x^2}{2k}$ (c) $\Delta t \le \frac{\Delta x^2}{4k}$ (d) $\Delta t \le \frac{2\Delta x^2}{k}$

Group – B

2. (a) The amount of mass transported by a pipe over a period of time can be computed using the equation

$$\int_{t_1}^{t_2} Q(t)c(t)dt \quad \text{where} \quad \begin{aligned} Q(t) &= 9 + 4\cos^2(t) \\ c(t) &= 5e^{-0.5t} + 2e^{0.15t} \end{aligned}$$

Calculate the total mass flow from 2 mins to 8 mins using Simpson's integration.

(b) Use trapezoidal rule to calculate the same and calculate the approximation error.

6 + 6 = 12

3. (a) The following data defines the sea-level concentration of dissolved oxygen in fresh water as a function of temperature:

T°C	0	8	16	24	32	40
O, ppm	14.621	11.843	9.870	8.418	7.305	6.413

Find out the oxygen concentration at 27°C using Lagrangian interpolation technique.

(b) Between Lagrangian method and Newton's method, which of the interpolation methods will provide a good stability and why?

9 + 3 = 12

B.TECH/CHE/5TH SEM/CHEN 3104/2017

Group – C

4. (a) For a linear system of equations $400x_1 - 201x_2 = 200$ and $401x_2 - 800x_1$ = comment on the condition of the system.

(b) Find out the solution for the following system using Gauss-Siedel iterative sc
$2x + 4y - z = 1$; $4x + y + z = -2$; $2x - 3y + 6z = 1$
Show atleast two iterations, with the initial guesses as '0'.

4 + 8

5. (a) The following system of equations is designed to determine concentr (the c's in g/m³) in a series of coupled reactors as a function of the a of mass input to each reactor (the right-hand sides in g/day),

$$15c_1 - 3c_2 - c_3 = 3300$$
$$-3c_1 + 18c_2 - 6c_3 = 1200$$
$$-4c_1 - c_2 + 12c_3 = 2400$$

Solve the system using LU decomposition method.

(b) For the previous problem of 5(a), determine how much the r mass input to reactor 3 must be increased to induce a 10 g/m³ r the concentration of reactor 1 in the above problem.

10 + 2

Group – D

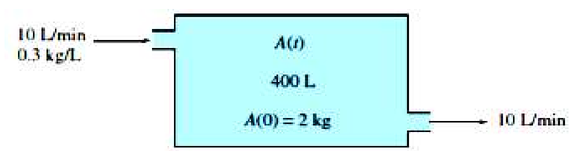
6. (a) Solve the following initial value problem with Heun's m

$$\frac{dy}{dt} = -\frac{2y}{t}$$

Use a step size of 0.5 and the initial values of y(0) = 3 to cal values at y(0.5) , y(1.0), y(1.5) ,y(2).

(b) Use Ralston's method to solve the same system as in 7(a) and co truncation error for y(1.0) and y(1.5).6 + 6 = 12

7. (a) Suppose a brine containing 0.3 kilogram (kg) of salt per liter (L into a tank initially filled with 400L of water containing 2 kg of the brine enters at 10 L/min, the mixture is kept uniform by st and the mixture flows out at the same rate. Find the mass of salt tank after 10 min (see Figure).



[*Hint:* Let A denote the number of kilograms of salt in the tank at t min after the process begins and use the fact that
rate of increase in A = rate of input - rate of exit.
Formulate the ODE in terms of concentration of A, C_A with all initial conditions. You can assume that the output concentration is same as the concentration of A, C_A in tank. Make sure that units on both sides of your equation are consistent.

(b) Solve the system in (a) using a second order RK method at steady state.

5 + 7 = 12

Group – E

8. (a) The governing equation and all the boundary conditions for a heat conduction problem is given as below.

$$\frac{\partial T}{\partial t} = 0.25\frac{\partial^2 T}{\partial x^2}$$

Subject to the boundary conditions

$$T(0,t) = 0$$

$$T(2,t) = 0$$

And initial condition: $T(x,0) = 2\sin(\pi x/2) - \sin(\pi x) + 4\sin(2\pi x)$

Choose 3 internal grid point along the length of the rod. Setup the finite difference equations for the heat conduction problem.

(b) Evaluate the temperature at the first time step in problem 8(a).

6 + 6 = 12

9. (a) A heated rod with uniform heat source can be modelled with the Poisson's equation as

$$\frac{d^2T}{dx^2} = -f(x) \quad \text{with } f(x) = -2.4x^2 + 12x$$

The rod is of length 10 cm. The temperature at the left end of the rod is 40^{o}C and at the right end is 200^{o}C. Setup the algebraic equations for the solution of temperature distribution using 2.5 cm divisions. Solve using Tridiagonal matrix algorithm.

(b) Setup the same problem assuming that the left end of the rod is connected to an oven from which heat is diffusing at a constant rate of 20 cal/(cm^2.s). The right end of the rod is connected to a heat sink at 50 ^{o}C. Be consistent with units when setting up your boundary conditions. The thermal conductivity of the rod is given to be 2 W/(cm K).

6 + 6 = 12

NUMERICAL METHODS OF ANALYSIS
(CHEN 3104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value after integrating $f(x)=0.2+25x-200x^2+675x^3-900x^4+400x^5$ within the limit 0 and 0.8 using trapezoidal rule is
(a) 0.1728 (b) 0.1628 (c) 0.1528 (d) 0.1928.

(ii) Given the two points [a,f(a)], [b,f(b)], the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by
(a) $f_1(x) = \frac{x+b}{b-a}f(a) + \frac{x+a}{b-a}f(b)$
(b) $f_1(x) = \frac{x-b}{b-a}f(a) + \frac{x-a}{b-a}f(b)$
(c) $f_1(x) = \frac{x+b}{b-a}f(b) + \frac{x+a}{b-a}f(a)$
(d) $f_1(x) = \frac{x-b}{b-a}f(b) + \frac{x-a}{b-a}f(a)$

(iii) For the given distributed data the value of $\Delta^3 y_0$ is

x	3.6	3.65	3.7	3.75
y	36.6	38.5	40.4	42.5

(a) 0.095 (b) 0.007 (c) 1.872 (d) 0.123.

(iv) In the Gauss elimination method for solving a system of linear algebraic equations, triangularization leads to
(a) diagonal matrix (b) lower triangular matrix
(c) upper triangular matrix (d) singular matrix.

(v) The convergence of which of the following method is sensitive to starting value?
(a) false position method (b) Gauss Siedel method
(c) Newton-Raphson method (d) all of these.

(vi) For a matrix $A = \begin{bmatrix} 2 & 3 & -7 \\ 5 & 4 & -2 \\ 7 & -3 & 6 \end{bmatrix}$, the infinity norm $\|A\|$ is equal to
(a) 12 (b) 11 (c) 16 (d) 39.