9. (a) Find an optimal sequence for the following sequencing problems of 4 jobs and 5 machines, when passing is not allowed. Its processing time is given below.

Jobs	Machines				
	M1	M2	M3	M4	M5
A	7	5	2	3	9
B	6	6	4	5	10
C	5	4	5	6	8
D	8	3	3	6	6

Find the total elapsed time and the idle time for each machine.

(b) In Birth and Death model show that, expected number of customers in the system is $\rho/(1-\rho)$ where $\rho = \lambda/\mu$, λ = mean arrival rate of customer and μ = mean service rate of customer.

6 + 6 = 12

OPTIMIZATION TECHNIQUES
(MCAP 1204)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The solution of a transportation problem with m rows and n columns is feasible if the number of positive allocations are
(a) m+n (b) mn (c) m+n-1 (d) m+n+1.

(ii) The possible number of basic solutions in a system of m equations in n unknowns are
(a) ${}^nC_m$ (b) mn
(c) (m+n)! (d) none of these.

(iii) The role of artificial variable in the simplex method is
(a) to aid in finding an initial solution
(b) to find the optimum dual prices in the final simplex table
(c) to start phases of simplex method
(d) all of these.

(iv) In an assignment problem, 'k' is the maximum number of zeros which can be assigned, then the minimum number of lines which will cover all the zeros is
(a) k (b) k-1 (c) k+1 (d) 2k.

(v) For a two person game of A and B, the pay-off matrix is given for B. The optimum strategies are
(a) minimax for A,maximin for B
(b) maximin for A , minimax for B
(c) minimin for A, maximin for B
(d) maximin for A, minimin for B.

(vi) In a PERT network the starting vertex is a
(a) burst node (b) merge node
(c) root (d) none of these.

(vii) CPM is ____ method
(a) probabilistic (b) deterministic
(c) event oriented (d) all of these.

(viii) If an activity has zero slack, it implies that
(a) it lies on the critical path
(b) it is a dummy activity
(c) the project is progressing well
(d) none of these.

(ix) The calling population is assumed to be infinite when
(a) arrivals are independent of each other
(b) capacity of the system is infinite
(c) service rate is faster than arrival rate
(d) all of these.

(x) A basic solution of the system of equation:
$2x + y - z = 2$; $3x + 2y - z = 3$ is
(a) (1, 1, 0) (b) (1, 1, 1)
(c) (1, 0, 0) (d) none of these.

Group – B

2. (a) Solve the following linear programming problem using graphical method and find the optimal solution if exists.

Mininize $Z = X_1 + X_2$
Subject to
$5X_1 + 9X_2 \leq 45$
$X_1 + X_2 \geq 2$
$X_2 \geq 4$
and $X_1, X_2 \geq 0$

(b) A pharmaceutical company produces two pharmaceutical products, A and B. Production of both products requires the same processes, I and II. The production of B results also in a by-product C at no extra cost. The product A can be sold at a profit of Rs. 3 per unit and B at a profit of Rs. 8 per unit. Some of this by-product can be sold at a unit profit of Rs. 2, the remainder has to be destroyed and the destruction cost is Rs. 1 per unit. Forecasts show that only up to 5 units of C can be sold. The company gets 3 units of C for each unit of B produced.

7. Consider the project with activity and duration given as follows:

Activity	Immediate predecessor	Duration (Days)
A	–	4
B	–	10
C	–	12
D	A	8
E	A	6
F	B, D	10
G	C	8
H	E	10
I	F, G	8
J	G, H	10
K	I, J	6

i. Draw the network diagram for the project
ii. Identify the critical path.
iii. What is the project duration?
iv. Find out the total float associated with each activity.

(4 + 4 + 1 + 3) = 12

Group – E

8. (a) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the interarrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average of 36 minutes. calculate
(i) expected queue size (in length).
(ii) probability that the queue size exceeds 10 if the input of trains increases to an average of 33 per day, what will be the change in (i) and (ii).

(b) Find the sequence that minimizes the total time required in performing the following jobs on 3 machines in the order A B C. Processing times are given in the following table:

Job	:	1	2	3	4	5
Machine A	:	8	10	6	7	1
Machine B	:	5	6	2	3	4
Machine C	:	4	9	8	6	5

Also find the total minimum elapsed time.

(3 + 4) + 5 = 12

The manufacturing times are 3 hours per unit for A on process I and II, respectively, and 4 hours and 5 hours per unit for B on process I and II, respectively. Because the product C results from producing B, no time is used in producing C. The available times are 18 and 21 hours of process I and II, respectively.
Formulate this problem as an LP model to determine the quantity of A and B which should be produced, keeping C in mind, to make the highest total profit to the company.

5 + 7 = 12

3. (a) Use penalty (Big M) method to solve the following linear programming problem.
Minimize $Z = 600X_1 + 500X_2$
subject to the constraints
$2X_1 + X_2 \geq 80$
$X_1 + 2X_2 \geq 60$
$X_1, X_2 \geq 0$

(b) Find the dual of the following linear programming problem
Minimize $Z = 2X_1 + 7X_2 + 5X_3$
subject to the constraints
$2X_1 + 5X_2 + 7X_3 \leq 17$
$3X_1 + 2X_2 + 5X_3 = 13$
$5X_1 + 3X_2 + X_3 \leq 9$
$X_1, X_3 \geq 0$, X_2 is unrestricted in sign.

7 + 5 = 12

Group – C

4. (a) A company has four manufacturing plants P_1, P_2, P_3, P_4 and five warehouses W_1, W_2, W_3, W_4, W_5. Each plant manufactures the same product, which is sold at different prices in each warehouse area. The capacities of the plants are 100, 200, 120 and 80 respectively. Demands of warehouses are 80, 120, 150, 70 and 90 respectively. The profit (Rs.) per unit product taken from different plant to different warehouses are given bellow:

Plant \ Warehouse	W_1	W_2	W_3	W_4	W_5
P_1	6	4	6	4	8
P_2	6	6	4	10	8
P_3	11	10	7	14	7
P_4	15	12	6	14	9

i. Formulate this problem as a transportation problem in order to maximize profit.
ii. Find the initial solution using Vogel's approximation method.
iii. Test for optimality and find the optimal solution.

(b) Give the economic interpretation of u_i's and v_j's in Transportation Problem.

(3 + 4 + 3) + 2 = 12

5. (a) A company has five jobs to be done. The adjacent matrix shows the return in rupees on assigning the machines. Assign the five jobs to five machines to maximize the expected return.

	Job1	Job2	Job3	Job4	Job5
P	32	38	40	28	40
Q	40	24	28	21	36
R	41	27	33	30	37
S	22	38	41	36	36
T	29	33	40	35	39

(b) "An assignment problem is a special case of transportation problem"— justify the statement.

8 + 4 = 12

Group – D

6. (a) What do you mean by payoff matrix and saddle point? Explain with example.

(b) Explain the principal of dominance in game theory and solve the following game

Player A	**Player B**				
	B_1	B_2	B_3	B_4	B_5
A_1	2	4	3	8	4
A_2	5	6	3	7	8
A_3	6	7	9	8	7
A_4	4	2	8	4	3

(2 + 3) + (2 + 5) = 12