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Geometric nonlinear free vibration of axially functionally graded non-uniform beams supported on elastic foundation

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Abstract: In the present study non-linear free vibration analysis is performed on a tapered Axially Functionally Graded (AFG) beam resting on an elastic foundation with different boundary conditions. Firstly the static problem is carried out through an iterative scheme using a relaxation parameter and later on the subsequent dynamic problem is solved as a standard eigen value problem. Minimum potential energy principle is used for the formulation of the static problem whereas for the dynamic problem Hamilton's principle is utilized. The free vibrational frequencies are tabulated for different taper profile, taper parameter and foundation stiffness. The dynamic behaviour of the system is presented in the form of backbone curves in dimensionless frequency-amplitude plane.

Keywords: Axially functionally graded beam; Elastic foundation; Large amplitude; Energy principles; Geometric nonlinearity; Backbone curve

Nomenclature

α	taper parameter
δ	variational operator
$[K]$	stiffness matrix
$[M]$	mass matrix
$[K_s]$	static stiffness matrix
$\{f\}$	load vector
π	total potential energy of the system
τ	time coordinate
ε_x^b, ε_x^s	axial strains due to bending and stretching respectively
ξ	normalized axial coordinate
b	width of the beam
L	length of the beam
ng	number of Gauss points
nw, nu	number of constituent functions for w and u respectively
P	magnitude of uniformly distributed load
T	kinetic energy of the system
U	strain energy stored in the system
u	displacement field in x-axis
V	potential energy of the external forces
w	displacement field in z-axis
ω_1	first natural frequency
ω_{nl}	nonlinear frequency parameters
ϕ_i	set of orthogonal functions for w
ψ_i	set of orthogonal functions for u
ρ_0	density of the beam material at the root
A_0	cross-sectional area of the beam at the root
c_i	unknown coefficients for static analysis
d_i	unknown coefficients for dynamic analysis
E_0	elastic modulus of the beam material at the root
I_0	moment of inertia of the beam at the root
t_0	thickness of the beam at the root
$w_{\max}$	maximum deflection of the beam

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1 Introduction

Various important engineering structures can be practically modeled as beams on elastic foundation. The key issue for such type of problem is to analyze the interaction between the foundation and the structural element at the interface. In most cases, certain idealizations are incorporated replacing the foundation by simple models, such as incorporating a series spring elements. The stiffness of the linear springs describes the behavior of the elastic foun-