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Natural Frequency and Mode Shapes of Exponential Tapered AFG Beams on Elastic Foundation

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Abstract. A displacement based semi-analytical method is utilized to study non-linear free vibration and mode shapes of an exponential tapered axially functionally graded (AFG) beam resting on an elastic foundation. In the present study geometric nonlinearity induced through large displacement is taken care of by non-linear strain-displacement relations. The beam is considered to be slender to neglect the rotary inertia and shear deformation effects. In the present paper at first the static problem is solved through an iterative scheme using a relaxation parameter and later on the subsequent dynamic analysis is carried out as a standard eigen value problem. Energy principles are used for the formulation of both the problems. The static problem is solved by using minimum potential energy principle whereas in case of dynamic problem Hamilton's principle is employed. The free vibrational frequencies are tabulated for exponential taper profile subject to various boundary conditions and foundation stiffness. The dynamic behaviour of the system is presented in the form of backbone curves in dimensionless frequency-amplitude plane and in some particular case the mode shape results are furnished.

Introduction

The ceramic-metal gradation technique in structural materials initiated by the Japanese material scientists in Sendai indicated the beginning of an era of advanced class of materials known as Functionally Graded Materials (FGMs). Since then various possibilites of using FGMs for different structural applications have been explored [1]. These FGMs, first invented in 1984, are increasingly being utilized in aerospace, aircraft, automobile and defense industrial applications. Their popularity is mainly due to their ability of mitigating the problem caused by the sudden change of thermo-mechanical properties as in the case of laminated composites [2,3]. Since the behavior of structural members made up with FGMs are of critical importance, they have received great attention from researchers in the last decades.

For functionally graded beams, gradient variation may be orientated in the cross-section or/and in the axial direction. For the former, there have been a large number of researches devoted to understand the static, buckling and dynamic behavior of the functionally graded structural components. Various methods are employed, including analytical method, semi-analytical method and numerical methods such as finite element method (FEM), differential quadrature method (DQM) and harmonic differential quadrature method (HDQM), various meshless methods, and quadrature element method (QEM) etc. Su *et al.* [4] formulated the dynamic stiffness method to investigate the free vibration behaviour of functionally graded beams. In their analysis derivation of the governing differential equations of motion was done utilizing Hamilton's principle. Kodali *et al.* [5] used displacement field based on higher order shear deformation theory to represent the static behavior of functionally graded metal–ceramic (FGM) beams under ambient temperature. Jin and Wang [6] established an N-node novel weak form quadrature functionally graded (FG) beam element based on the classical beam theory (CBT) and differential quadrature (DQ) rule. Yaghoobi