9. (a) Find the general solution of the following partial differential equation
$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=\cos(2x+y)$$

(b) Solve the following partial differential equation $z(xp-yq)=y^2-x^2$

6 + 6 = 12

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of $f(3)$, where $f(a)=2\int_C \frac{2z^2-z-2}{(z-a)}dz, C:|z|=2.5$ *is*

(a) 0 (b) 1 (c) πi (d) -1.

(ii) If the function $f(z)$ is not analytic at $z=0$, then $z=0$ is called

(a) an ordinary point of $f(z)$ (b) a singular point of $f(z)$

(c) zero of $f(z)$ (d) none of these.

(iii) If $F(s)$ is the Fourier Transform of $f(x)$, $a \neq 0$ then $F\{f(ax)\}$ is

(a) $\frac{1}{a}F(s/a)$ (b) $-\frac{1}{a}F(s/a)$

(c) $\frac{1}{|a|}F(s/a)$ (d) $-\frac{1}{|a|}F(Sa)$

(iv) If the function $f(x)=x, -\pi<x<\pi$ is represented by a Fourier series as $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$, then $a_n=$

(a) 0 (b) 1 (c) 2 (d) ½.

(v) The residue of $\frac{z^2}{z^2+a^2}$ at $z=ia$ is

(a) $-\frac{1}{2}ia$ (b) $\frac{1}{2}ia$ (c) ia (d) a

(vi) The solution of the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$ is
(a) $P_n(x)$ (b) $P_1(x)$ (c) $P_2(x)$ (d) $P_3(x)$

(vii) $J_{\frac{1}{2}}(x)$ is given by
(a) $\sqrt{\frac{2\pi}{x}}\sin x$ (b) $\sqrt{\frac{2\pi}{x}}\cos x$ (c) $\sqrt{\frac{\pi}{2x}}\cos x$ (d) $\sqrt{\frac{2}{\pi x}}\sin x$

(viii) P. I. of $\left(D^2 + 4DD' - 5D'^2\right) = \sin(2x+3y)$ is
(a) $2\sin(2x+3y)$ (b) $\frac{3}{4}\sin(3x+2y)$
(c) $\frac{1}{17}\sin(2x+3y)$ (d) $\sin(2x+3y)$

(ix) The solution of the p. d. e $p - q = 1$ is
(a) $z = ax + (1-a)y$ (b) $z = ax$
(c) $z = ax + (a-1)x + c$ (d) $z = c$

(x) The Fourier series of an even function contains only
(a) cosine terms (b) sine terms
(c) sine and cosine terms (d) none of these.

Group – B

2. (a) Use the definition of limit to prove that $\underset{z\to 1-i}{Lt}\left\{z + i\left(z+\bar{z}\right)\right\} = 1 + i$.

(b) Determine whether the function $v = -e^{-x}\sin y$ is harmonic. If harmonic, then find the analytic function $f(z) = u + iv$ in terms of z.

6 + 6 = 12

3. (a) State Cauchy's Integral formula and if $f(z) = 5z^2 - 4z + 3$, then show that
$\int_\tau \frac{f(z)}{z+3i}dz = -\pi(24+84i)$, where τ is the ellipse $16x^2 + 9y^2 = 144$.

(b) Use Residue theorem to evaluate $\int_{|z|=\frac{1}{2}} \frac{(z+1)}{z^3 - 2z^2}\,dz$

6 + 6 = 12

Group – C

4. (a) Find the Fourier series expansion of $f(x) = x + x^2$ on $[-\pi, \pi]$.
Hence, deduce that
$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$.

(b) Find the Fourier sine transform of the function $e^{-|x|}$. Hence prove that $\int_0^\infty \frac{x\,si}{(1+x^2)}dx = \frac{\pi}{2}e^{-m},\; m > 0$.

6 + 6 = 12

5. (a) Find the Fourier transform of $f(t)$ defined by
$$f(t) = \begin{cases} e^{-at} & for\ t > 0 \quad (a>0) \\ 0 & for\ t < 0 \end{cases}$$
If $F\{h(t)\} = \frac{1}{(1-is)^2}$ and $h(t) = 0$ for $t < 0$, find $h(t)$ using convolution theorem.

(b) Find the Fourier inverse transform of the function $F(s) = \frac{1}{s^2+4s+13}$.

7 + 5 = 12

Group – D

6. (a) Solve in series $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ about the point $x = 0$.

(b) Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

7 + 5 = 12

7. (a) Solve by Frobenius method: $2x^2y'' + xy' - (x+1)y = 0$.

(b) Prove the following recursion relation for Bessel's function using generating function
$xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$

8 + 4 = 12

Group – E

8. (a) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

(b) Form the partial differential equations (by eliminating the arbitrary functions) from $f(x^2+y^2, z - xy) = 0$.

6 + 6 = 12