(vi) If algorithm A has running time $7n^2 + 2n + 3$, and algorithm B has running time $2n^2$, then
(a) both has same asymptotic time complexity
(b) A is asymptotically greater
(c) B is asymptotically greater
(d) None of the above.

(vii) Optimal substructure property is exploited by
(a) dynamic Programming (b) greedy method
(c) both (a) and (b) (d) none of above.

(viii) The Worst case occurs in linear search algorithm when
(a) item is somewhere in the middle of the array
(b) item is not in the array at all
(c) item is the last element in the array
(d) item is the last element in the array or is not there at all.

(ix) Which of the following algorithm design techniques is used in the quick sort algorithm?
(a) Dynamic programming (b) Backtracking
(c) Divide and conquer (d) Greedy method.

(x) Which one of the following is true
(a) All NP hard problems are NP complete
(b) All NP complete problems are NP hard
(c) Some NP complete problems are NP hard
(d) None of these.

Group – B

2. (a) State master theorem.

(b) (i) Give asymptotic tight bounds for the following (using master method)

$$T(n) = 9T(n/4) + n^2$$

(ii) Prove that $(n+1)^2 = O(n^2)$

(iii) Prove that $2n^2 - 5n - 100 = \Theta(n^2)$

(c) Let T(n) be the running time of the algorithm. The recurrence relation of T(n) is given below:

$$T(n) = \begin{cases} c & \text{if } n=0 \\ 2T(n-1)+c & \text{if } n>0 \end{cases}$$

Show that $T(n) = \Theta(2^n)$.

3 + (2 + 2 + 2) + 3 = 12

3. (a) How would you modify Strassen's algorithm to multiply n × n matrices in which n is not an exact power of 2? Show that the resulting algorithm runs in time $\Theta(n \log 7)$.

(b) Illustrate the operation of HEAPSORT on the array A = {5, 13, 2, 25, 7, 17, 20, 8, 4}. Also show that the worst-case running time of HEAPSORT is $\Omega(n \log n)$.

(c) Show that log(n!) is approximately equal to n log n – n log e + O (1) by using the fact that the function log k is monotonic and bounded below by

$$\int_{k-1}^{k} \log x \, dx$$

(2 + 2) + (2 + 3) + 3 = 12

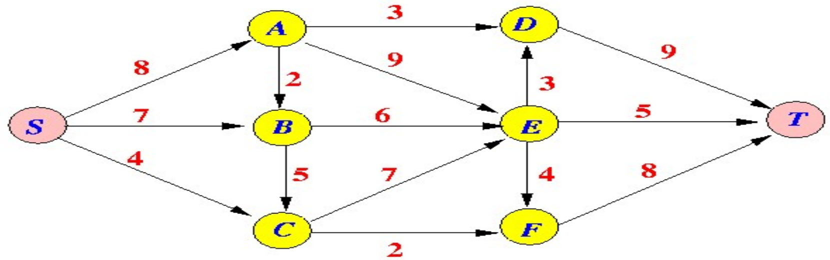
Group – C

4. (a) Write an algorithm of Depth First Search and deduce its' time complexity.

(b) A Sudoku puzzle is defined as a logic-based, number-placement puzzle. The objective is to fill a 9 × 9 grid with digits in such a way that each column, each row, and each of the nine 3 × 3 grids that make up the larger 9 × 9 grid contains all of the digits from 1 to 9. Solve this puzzle using Graph Coloring.

(4 + 2) + 6 = 12

5. (a) Write the Max flow- Min cut theorem. Apply the same for the following network to find the min-cut using max-flow.



(b) What is bidirectional search? Write an algorithm to implement the same. What is the time complexity of the same?

(2 + 4) + (1 + 3 + 2) = 12

Group – D

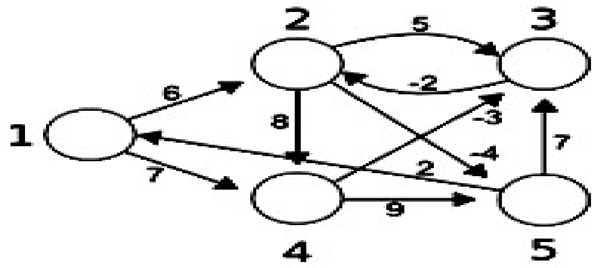
6. (a) Using Dynamic Programming to solve the Travelling Salesman Problem for the following graph(Adjacency matrix is given)

i/j	1	2	3	4	5
1	0	2	1	6	1
2	1	0	4	4	2
3	5	3	0	1	5
4	4	7	2	0	1
5	2	6	3	6	0

(b) Compare Prim's and Kruskal's algorithm and deduce time complexity for both.

7 + (2 + 3) = 12

7\. (a) Write down the Bellman-Ford's algorithm to find the all pair shortest path. Find its complexity. Apply the same to find the all pair shortest paths for the following graph:



(b) Given the weight vector (3, 2, 8, 4, 6, 1, 2) and the profit vector (11, 10, 9, 16, 8, 7, 9) and a knapsack of capacity 16, find at least three feasible solution including optimal one for the knapsack problem of seven object.

(3 + 2 + 3) + 4 = 12

Group – E

8\. (a) What is branch & bound technique? Write the Least-cost search method to find the solution of the 15 puzzle problem. Will this search method guarantee solution of the problem for all instances?

(b) Give an example of NP-hard decision problem which is not NP-complete. Also give reasons for the same.

(2 + 4 + 1) + 5 = 12

9\. (a) Show that chromatic number decision problem is NP-complete.

(b) Is it possible to design a polynomial time absolute approximation algorithm for planar graph coloring? If no, why it's not possible, if yes how is it possible?

6 + (2 + 4) = 12

B.TECH/IT/4TH SEM/INFO 2202/2017

DESIGN & ANALYSIS OF ALGORITHMS (INFO 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1\. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following basic algorithms can be used to most efficiently determine the presence of a cycle in a given graph
(a) minimum cost spanning tree algorithm
(b) Ford-Fulkerson's algorithm
(c) breadth-first search algorithm
(d) depth-first search algorithm.

(ii) O-notation provides an asymptotic
(a) upper bound (b) lower bound
(c) light bound (d) none of these.

(iii) The total running time of knapsack problem for a simple approach is
(a) O(n) (b) O(log n)
(c) $O(2^n \log n)$ (d) $O(2^n)$.

(iv) To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is:
(a) Queue (b) Stack (c) Heap (d) B-Tree.

(v) In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of time complexity by
(a) Dijkstra's algorithm starting from S
(b) Warshall's algorithm
(c) Performing a DFS starting from S
(d) Performing a BFS starting from S.