

$T = \{ 0, 1, 2, 3, 4 \}$ ,  $S$  is the start symbol, and the productions are as follows:

$S \rightarrow A, S \rightarrow B, S \rightarrow 4, A \rightarrow 012S, B \rightarrow 012S3S.$

Show that the grammar  $G$  is ambiguous.

- (b) Provide a Type 2 (context-free) grammar for the following language  $L$  over the input alphabet  $\{ 0, 1 \}$ :

$L = \{ \alpha \mid \text{the string } \alpha \text{ has more 0's than 1's} \}$

**(2 + 4) + 6 = 12**

7.  $G = (V, T, S, P)$  is a Type 2 grammar, with  $V = \{ S, A, B, 0, 1, 2, 3, 4 \}$ ,  $T = \{ 0, 1, 2, 3, 4 \}$ ,  $S$  as the start symbol and the following five production rules:

$S \rightarrow A, S \rightarrow B, S \rightarrow 4, A \rightarrow 012S, B \rightarrow 012S3S.$

Let  $L$  be the language generated by  $G$ .

- (i) Design a pushdown acceptor (pda)  $M$  that accepts  $L$ .  
 (ii) Describe the steps by which the input string '012430124' is accepted by  $M$ . Also explain why the input string '0124340124' is not accepted by  $M$ .

**(6 + 3 + 3) = 12**

### Group - E

- 8.(a) Two positive integers  $m$  and  $n$  are written on a Turing machine tape in unary notation. The integer  $m$  is to the left of the integer  $n$  on the tape, and a single blank cell separates the two integers. At start the read/write head is positioned on the leftmost 1 of  $m$ . Give the state transition diagram of a Turing machine  $M$  that will halt on the leftmost 1 of  $m$  if  $m \geq n$ , and will halt on the leftmost 1 of  $n$  if  $m < n$ . Clearly state any assumptions made.

- (b) Describe how  $M$  operates on the following two sets of inputs:

(i)  $m = 5, n = 2$ ; (ii)  $m = 2, n = 3$ .

**8 + 4 = 12**

- 9.(a) Explain what is meant by a 'Universal Turing Machine'. Can every effective procedure be implemented by such a machine assuming the input is appropriately supplied?

- (b) What is the 'Halting Problem' for Turing Machines? What do we mean when we say that the Halting Problem for Turing machines is unsolvable?

**6 + 6 = 12**

## THEORY OF COMPUTATION

(CSEN 5201)

Time Allotted: 3 hrs

Full Marks: 70

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and Any 5 (five) from Group B to E, taking at least one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

### Group – A

#### (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

- (i) It is known that the deterministic finite state acceptor (dfa)  $M$  recognizes all the binary strings (and only those) in the regular expression  $0^*1011^*$ . Then  $M$  must have at least  $n$  states, where  $n$  equals to  
 (a) 2 (b) 3 (c) 4 (d) 5.
- (ii) If  $L_1$  and  $L_2$  are two context free languages. Then  $L_1 \cap L_2$   
 (a) need not be regular but can be context free  
 (b) need not be context free but can be context sensitive  
 (c) is always regular  
 (d) is always context free.
- (iii) If it is known that the total number of strings of length  $\leq 4$  using  $n$  terminal symbols exceeds 300, then  $n$  must be at least  
 (a) 2 (b) 3 (c) 4 (d) 5.
- (iv) A positive integer  $m$  when written in binary notation (with no leading zeroes) has a length of 21 binary digits. When  $m$  is written in decimal notation (with no leading zeroes), its length will be exactly  $n$  decimal digits, where  $n$  is  
 (a) 4 (b) 5 (c) 6 (d) 7.
- (v) The Pumping Lemma for Context-Free Languages can be used to prove that a given language  $L$   
 (a) is not Type 0 (b) is not Type 1  
 (c) is not Type 2 (d) is not Type 3.

- (vi) A machine  $M$  has been designed that, given a positive integer  $n$  as input, accepts  $n$  if and only if  $n$  is a perfect square. Then which one of the following alternatives is false?
- $M$  cannot be a deterministic finite state acceptor (dfsa)
  - $M$  cannot be a deterministic pushdown acceptor (dpda)
  - $M$  cannot be a non-deterministic pushdown acceptor (ndpda)
  - $M$  cannot be a Turing machine.
- (vii) If the original nondeterministic finite state automata (ndfsa) has  $n$  states, the corresponding deterministic finite state automata (dfsa) cannot have
- more than  $n$  states
  - more than  $2n$  states
  - more than  $2^n$  states
  - more than  $(n-1)$  states.
- (viii) Which one of the following languages cannot be accepted by a non-deterministic pushdown acceptor (ndpda)?
- $\{0^m 1^n \mid 0 < m, 0 < n\}$
  - $\{0^m 1^n 0^m \mid 0 < m, 0 < n\}$
  - $\{0^m 1^n 0^m \mid 0 < m\}$
  - $\{0^m 1^n 0^r \mid 0 < m, 0 < n, 0 < r\}$ .
- (ix) Which of the following pairs of machines given below *do not* have equal computing power?
- Deterministic and Nondeterministic finite state automata
  - Deterministic and Nondeterministic pushdown automata
  - Deterministic and Nondeterministic Turing Machine
  - Multi tape Turing machine and Universal Turing machine.
- (x) We are supplied as input the state tables of two deterministic finite state acceptors (dfsas)  $M_1$  and  $M_2$ . The input alphabet is  $\{0, 1\}$ . We want to find out whether the languages  $L(M_1)$  and  $L(M_2)$  are identical sets, i.e., whether they contain exactly the same strings. Then which statement below is true?
- There is a dfsa that will solve the problem.
  - There is an ndpda that will solve the problem but not a dfsa.
  - There is a Turing machine that will solve the problem but not an ndpda.
  - There is no effective procedure for solving the problem.

**Group – B**

2. Consider the set of strings  $L_1$  on the alphabet  $\{0, 1\}$  that contains *Even* number of 0's. Consider another set of strings  $L_2$  on the alphabet  $\{0, 1\}$  that contains *Odd* number of 1's.

- Design a deterministic finite state acceptor (dfsa)  $M_1$  that will accept  $L_1 \cap L_2$ . Show both the state table and the state transition diagram of  $M_1$ .
- Explain the design in brief with the help of suitable examples.

**(8 + 4) = 12**

3. (a) A non-deterministic finite state acceptor (ndfsa)  $M_1$  has the state table shown below.  $S$  is the start state, and  $B$  and  $C$  are goal states. Convert  $M_1$  to an equivalent deterministic finite state acceptor (dfsa)  $M_2$  and clearly mark the start and goal states.

	0	1
$\rightarrow S$	B,C	S,A
A	A,C	---
*B	---	A
*C	C	C

- Minimize the number of states in  $M_2$  designed above and get a reduced dfsa  $M_3$ .

**8 + 4 = 12****Group – C**

- (a) Construct a non-deterministic finite state acceptor (ndfsa)  $M$  on the input alphabet  $\{0, 1, 2\}$  that accepts a string  $\alpha$  if and only if  $\alpha$  is contained in the regular expression  $0^* 1^* 2^*$ .
- (b) Construct a non-deterministic finite state acceptor (ndfsa)  $N$  on the input alphabet  $\{0, 1, 2\}$  that accepts a string  $\alpha$  if and only if  $\alpha$  is *not* contained in the regular expression  $(01 + 2)^*$ .

**5 + 7 = 12**

- (a) Use the Pumping Lemma for Regular Languages, to show that the language  $L = \{0^m 1^n 0^m 1^n \mid m > 5, n > 10\}$  is not regular.
- (b) Prove that the Regular Languages are closed under union, intersection, concatenation and star closure operation.

**6 + 6 = 12****Group - D**

- (a) When is a Type 2 (context-free) grammar said to be ambiguous? Consider the grammar  $G = (V, T, S, P)$  where  $V = \{S, A, B, 0, 1, 2, 3, 4\}$ ,