(vi) A series u_n is said to be convergent if and only if _____ for all n

(a) $\left|\frac{u_n}{u_{n+1}}\right| < 1$ (b) $\left|\frac{u_n}{u_{n+1}}\right| > 1$

(c) $\left|\frac{u_n - u_{n+1}}{u_{n+1}}\right| > 0$ (d) both (b) and (c).

(vii) Laplace Transformation method is useful for solving PDEs because it
(a) reduces the number of variables
(b) converts PDEs into ODEs
(c) converts PDEs into algebraic equations
(d) can be solved numerically.

(viii) At the point where boundary layer separates
(a) $\tau_w = 0$ (b) $\tau_w = \infty$
(c) Normal stress = ∞ (d) The flow is irrotational.

(ix) δ/l for a boundary layer is proportional to
(a) 1/Re (b) Re (c) $1/\sqrt{Re}$ (d) $Re^{3/2}$.

(x) For large values of Reynolds number the disturbance thickness (δ) of a boundary layer approaches to
(a) 0 (b) ∞
(c) the length of the plate (*l*) (d) 0.664 *l*.

Group – B

2. (a) In a batch reactor, two consecutive first order reactions are taking place $A \xrightarrow{k_1} B \xrightarrow{k_2} C$. The mass balance leads to a set of linear ODEs describing concentration of A as C_A, concentration of B as C_B and concentration of C as C_C. Determine the solution using eigen problem formulation and show that for $k_2 >> k_1$, the system approaches to pseudo steady state condition for component B i.e. $C_B \cong \frac{k_1 C_A}{k_2}$.

(b) Find out the eigen value and the eigen vector for the matrix given as, $\begin{bmatrix} 2 & 1 \\ -4 & -3 \end{bmatrix}$.

9 + 3 = 12

3. (a) The following system of equations are designed to determine concentrations (the c's in g/m³) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right hand side in g/day)

$15c_1 - 3c_2 - c_3 = 3800$
$-3c_1 + 18c_2 - 6c_3 = 1200$
$-4c_1 - c_2 + 12c_3 = 2350$

c_1, c_2 and c_3 are concentrations in reactors 1, 2 and 3 respectively.

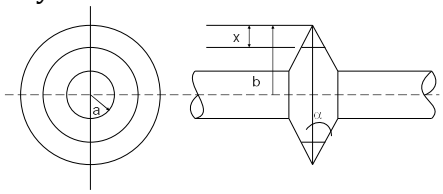
How much will the concentration in reactor 3 be reduced if the rate of mass input to reactors 1 and 2 is reduced by 500 and 250 g/day, respectively?

(b) "In a three stage separator unit, if after mass balance, it is seen that the rank of the coefficient matrix is equal to 2 then the degrees of freedom for the system is equal to 1." - Justify the correctness of the statement.

9 + 3 = 12

Group – C

4. (a) The shape of the cooling fin is shown in the following illustration, where the radius of the pipe (a) is 8 cm, the radius of the rim of the fin (b) is 20 cm, and the coordinate x (m) is measured inwards from the rim of the fin. There are two natural origins for the coordinate, but since the temperature distribution in the vicinity of the pipe axis is of no interest, the origin is taken on the rim instead. Assuming that the fin is thin, temperature variations normal to the central plane of the fin will be neglected. The thermal conductivity of the fin (k) is 380 W/m°C, and the surface heat transfer coefficient (h) is 12 W/m²°C. Calculate the rate of removal of heat by the fin.



(b) Show that $x\frac{d}{dx}J_k(\alpha x) = kJ_k(\alpha x) - \alpha x J_{k+1}(\alpha x)$, where $J_k(x)$ is the Bessel function.

9 + 3 = 12

5. (a) A supply of hot air is to be obtained by drawing cool air through a heated cylindrical pipe. The pipe is 0.1 m diameter and 1.2 m long, and is maintained at a temperature T_w = 300°C throughout its length. The properties of air are:
Heat capacity (C_p) = 1000 J/kg°C
Thermal conductivity (k) = 0.035 W/m°C
Density (ρ) = 0.8 kg/m³
Flow rate (u) = 0.009 m³/s
Inlet temperature = 20°C
convective transfer coefficient (h) = $\frac{10}{\sqrt{x}}$ W/m²°C

(b) Show that for equal roots (c) of indicial equation the solution for an equation of type $x^2\frac{d^2y}{dx^2}+xF(x)\frac{dy}{dx}+G(x)y=0$, where $F(x)=\sum_{n=0}^{\infty}F_nx^n$; $G(x)=\sum_{n=0}^{\infty}G_nx^n$ is given by $y=Au(x,c)+B\left(\frac{\partial u(x,c)}{\partial c}\right)_c$, where, A and B are constants; u(x,c) is the solution; F(x) and G(x) are having a radius of convergence.

9 + 3 = 12

Group – D

6. (a) Find out the concentration profile in a slab in the y direction under unsteady state if the initial concentration throughout the slab is uniform at C_0 and one face at y=0 suddenly attains a concentration C_1. Mention the appropriate initial and boundary conditions.
Given $L^{-1}\left(\frac{1}{s}e^{-k\sqrt{s}}\right)=\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$

(b) Explain the concept of orthogonal functions giving a suitable example.

8 + 4 = 12

7. (a) A partial differential equation is given: $\partial y/\partial t = \beta\,\partial^2 y/\partial z^2$. How can you obtain a solution for y as y = f(t)g(z) by the method of separation of variables?

(b) Solve the above-mentioned equation by Laplace transformation method, given the conditions:
i) at t = 0, y = y_0
ii) at z = 0, y = y_1 Given: $L^{-1}\left(\frac{1}{s}e^{-k\sqrt{s}}\right)=\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$

7 + 5 = 12

Group – E

8. (a) Determine an expression for displacement thickness of a boundary layer. Explain physically the relationship between displacement thickness and disturbance thickness.

(b) Explain the concepts of positive and negative pressure gradients in connection to boundary layer formation and separation.

(5 + 3) + 4 = 12

9. (a) How are the Prandtl boundary layer equations applied for describing the boundary layer flow over a flat plate at zero incidence?

(b) Determine an expression for momentum thickness of a boundary layer.

8 + 4 = 12

B.TECH/CHE/6TH SEM/CHEN 3204/2017
MATHEMATICAL METHODS IN CHEMICAL ENGINEERING
(CHEN 3204)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For the matrix $\begin{bmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{bmatrix}$ the number of eigen values will be
(a) 2 (b) 1 (c) 3 (d) 0.

(ii) Which of the following statements are TRUE?
P. The eigenvalues of a symmetric matrix are real.
Q. The value of the determinant of an orthogonal matrix can only be +1.
R. The transpose of a square matrix A has the same eigenvalues as those of A.
S. The inverse of an 'n × n' matrix exists if and only if the rank is less than 'n'.
(a) P and Q only (b) P and R only
(c) Q and R only (d) P and S only.

(iii) The rank of a matrix is defined as the _____ number of _____ rows/columns in the matrix
(a) maximum, independent (b) minimum, independent
(c) maximum, dependent (d) minimum, dependent.

(iv) Convergence by Jacobi iteration as compared to convergence by Gauss-Siedel iteration is
(a) fast (b) slow
(c) same (d) same depending on relaxation.

(v) The Bessel function value of 0^{th} order near the origin is equal to _____
(a) 1 (b) 0 (c) ±∞ (d) none of these.