M.TECH/ECE/IT/VLSI/1ST SEM/MATH 5103/2016

ADVANCED MATHEMATICS
(MATH 5103)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $T: U \to V$ is any linear transformation from U to V then
(a) the Ker(T) is a subspace of U
(b) the Ker(T) is a subspace of V
(c) the range of T is a subspace of U
(d) none of these.

(ii) If S is a basis of a vector space V, then
(a) S is linearly dependent (b) S is linearly independent
(c) S does not generate V (d) none of these

(iii) Which of the following algorithms are greedy?
(a) Djkstra's Algorithm (b) Prim's Algorithm
(c) Kruskal's Algorithm (d) All of these.

(iv) The number of stationary points of the function f(x,y)= x+y is
(a) 0 (b) 1 (c) infinite (d) 2.

(v) If V and W are finite dimensional vector spaces over a field F and T: V→W is a linear mapping, then
(a) Nullity of T+ Rank of T= dim V (b) Nullity of T + Rank of T = dim W
(c) Nullity of T + Rank of T > dim V (d) none of these.

(vi) If V is a finite dimensional vector space and S is a basis of V, then
(a) Number of elements of S = dimV
(b) Number of elements of S > dimV
(c) Number of elements of S < dimV
(d) None of these.

(vii) The standard deviation of a uniformly distributed random variable between 0 and 1 is
(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{5}{\sqrt{12}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{7}{\sqrt{12}}$.

(viii) Number of edges of the complete bigraph $K_{3,4}$ is
(a) 10 (b) 11 (c) 12 (d) none of these.

(ix) Choose the correct statement
(a) path is an open walk
(b) every walk is a trial
(c) every trial is a path
(d) a vertex cannot appear twice in a walk.

(x) If A and B are two events with $P(A) = \frac{5}{6}$, $P(B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P(A \cap \overline{B}) = ?$
(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{1}{6}$ (d) none of these.

Group – B

2. (a) There are three coins in a box. One is two-headed coin, another is a fair coin and the third is biased coin that turns up heads 75% of the time. When one of the three coin is selected at random and flipped, it shows heads. What is the probability that it was the two headed coin.

(b) A defective die is thrown ten times independently. The probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that odd face appear in each of ten throws.

6 + 6 = 12

3. (a) A system has three possible states, 0, 1 and 2. Every hour it makes a transition to a different state, which is determined by a coin flip. For example, from state 0, it makes a transition to state 1 or state 2 with probabilities 0.5 and 0.5.
(i) Find the transition probability matrix.
(ii) Find the three step transition probability matrix.
(iii) Find the steady-state distribution of the Marcov Chain.

(b) A random variable X has the probability density function

$$f(x)=\begin{cases} e^{-x}, \; x\geq 0 \\ 0, \; otherwise \end{cases}$$

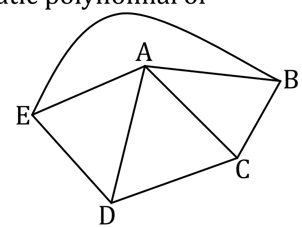
Find $E(X^2)$.

8 + 4 = 12

Group – C

4. (a) Prove that the chromatic number of a complete graph with n vertices (K_n) is n.

(b) Find the chromatic polynomial of



6 + 6 = 12

5. (a) Applicants a_1, a_2, a_3 and a_4 apply for five posts p_1, p_2, p_3, p_4 and p_5. The applications are done as follows: $a_1 \rightarrow \{p_1, p_2\}$, $a_2 \rightarrow \{p_1, p_3, p_5\}$, $a_3 \rightarrow \{p_1, p_2, p_3, p_5\}$ and $a_4 \rightarrow \{p_3, p_4\}$. Using graph theory find (i) whether there is any perfect matching of the set of applicants into the set of posts. If yes, find matching (ii) whether every applicant can be offered a single post.

(b) Let G be a regular graph, the degree of each of its vertices being 4. Determine the number of vertices of G if G determines 10 regions.

(c) Let G be a simple planar graph with less than 12 vertices. Prove that, G has a vertex whose degree ≤ 4.

6 + 3 + 3 = 12

Group – D

6. (a) Examine if the set S is a subspace of R^3, where S = $\{(x,y,z)\in R^3 : x+y+z=0\}$

(b) Prove that the set of vectors $\{(1,2,2),(2,1,2),(2,2,1)\}$ is linearly independent in R^3.

6 + 6 = 12

7. (a) Find KerT and its dimension, where T is the linear mapping $T:\Re^3 \rightarrow \Re^3$ defined by $T(x_1,x_2,x_3)=(x_1+x_2+x_3, 2x_1+x_2+2x_3, x_1+2x_2+x_3)$, $(x_1,x_2,x_3)\in\Re^3$.

(b) Determine the eigen values and eigen vectors of the matrix

$$A=\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}.$$

6 + 6 = 12

Group – E

8. (a) Use Lagrange's Multiplier Method to find all the critical points of $f(x,y,z)=x^2-y^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$. Determine the maxima and minima of f on the surface by evaluating f at the critical points.

(b) Form the initial simplex table corresponding to the following LPP:

Minimize $5x_1+7x_2$

Subject to $2x_1 + 9x_2 \leq 2$

$3x_1 - 2x_2 \leq 6$

$x_1 + 5x_2 \leq 4$

$x_1, x_2 \geq 0$

8 + 4 = 12

9. Solve the following LPP using Big-M Method:

Minimize $Z=6x_1+4x_2$

Subject to $2x_1+3x_2 \leq 30$

$3x_1+2x_2 \leq 24$

$x_1+x_2 \geq 3$

$x_1 \geq 0$ and $x_2 \geq 0$

12