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HERITAGE INSTITUTE OF TECHNOLOGY

M. Tech 1st Semester Examination. 2014 Session: 2014-2015

Discipline: ECE, VLSI, IT

Paper Code: MATH5103 Paper Name: Advanced Engineering Mathematics

Time Allotted: 3 hrs Full Marks: 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: 10 x 1=10

(i) The probability that the four children of a family have different birthdays (exclude leap year) is
(a) 0.9836 (b) 0.4735
(c) 0.8 (d) 0.757

(ii) A random variable X has the p.d.f. :
$f(x) = \frac{1}{4}, -2 < x < 2; f(x) = 0, elsewhere,$
then P(2X + 3 > 5) is
(a) 1 (b) ½ (c) ¼ (d) none of these

(iii) If $\lambda^4 - 3\lambda^3 - 6\lambda + 10$ is the characteristic polynomial of matrix A, then det A is equal to
(a) 10 (b) -3 (c) -6 (d) none of these

(iv) A ring of integers is an infinite integral domain which is
(a) a field (b) not a field
(c) skew field (d) None of these

(v) Let P_{99} be the collection of all polynomials with real coefficients of degree less than or equal to 99. The dimension of the real vector space P_{99} is:
(a) 99 (b) 98 (c) 100 (d) 0

(vi) Let W_1 and W_2 be two subspaces of a vector space V. Then
(a) $W_1 \cup W_2$ is a subspace of V (b) $W_1 \cup W_2$ is not a subspace of V
(c) $W_1 \cap W_2$ is not a subspace of V (d) none of these.

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(vii) A graph is Kuratowski's first graph with 5 vertices if it is

(a) Connected. (b) Planner.

(c) Complete. (d) None of the above.

(viii) If a binary tree has 21 pendant vertices, then the number of vertices of this binary tree is

(a) 20 (b) 42

(c) 41 (d) none of these

(ix) The chromatic number of a cycle of length 5 is

(a) 1 (b) 2

(c) 3 (d) 4

(x) If (a,b) is a saddle point of $f(x,y)$ then at (a,b), $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ should be

(a) both zero (b) one zero and the other non-zero

(c) both non-zero (d) none of these

Group – B

2.(a) There are two identical urns containing respectively 4 white, 3 red balls and 3 white, 7 red balls. An urn is chosen at random and a ball is drawn from it. If the ball drawn is white, what is the probability that it is from the first urn?

(b) The probability density function of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \le x \le 2$ where k is a constant. Find (i) the constant k and (ii) $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$

5+(3+4) = 12

3.(a) (i) How is the Transition Probability Matrix defined for a Markov Chain $\{X_0, X_1, X_2, \ldots\ldots, X_{n-1}, X_n, \ldots\ldots\}$?

(ii) There are two boxes numbered 1 and 2. There are d many balls distributed between these boxes (say i many in box 1 and the rest in box 2). Each time a ball is randomly selected and its box is changed. Let X_n be the number of balls in the first box after n such steps. Find out the state space (set of possible values of X_n) and the Transition Probability Matrix.

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(b) Consider a two state Markov Chain model for weather forecasting. The weather of a day might be dry (no rain during the day) or rainy (if it rains at least once during the day). Suppose if it rains today, then it will rain tomorrow with probability 0.7; and if it is dry today then it will rain tomorrow with probability 0.4. Calculate the probability that it will rain day-after-tomorrow if it rained yesterday, by applying Chapman-Kolmogorov equation. (2+4)+6 = 12

Group - C

4.(a) State and prove Euler's formula for planner graph.

(b) Let G be a connected regular planner graph such that the degree of each of its vertices being 3 and the graph determines 20 regions. Find the numbers of vertices of G. 6+6 = 12

5.(a) Prove that every planner graph is 6-colourable.

(b) What is chromatic polynomial of a graph with n vertices? Prove that the chromatic polynomial of a tree with n vertices is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$. 5+(2+5) = 12

Group - D

6.(a) State the necessary condition for a point (α, β) to be an extreme point of a function $f(x, y)$. Find the saddle points of the function $x^3 + y^3 - 3x - 12y + 20$.

(b) If x y z = c^3, a constant, using Lagrange's multiplier method, evaluate the minimum value of f(x,y,z) = xy + yz + zx. (2+4)+6 = 12

7.(a) Using simplex method to solve the following Linear Programming:

$$\text{Maximize } z = 4x_1 + 7x_2$$
$$\text{subject to} \quad 2x_1 + x_2 \le 1000$$
$$10x_1 + 10x_2 \le 6000$$
$$2x_1 + 4x_2 \le 2000,\ x_1, x_2 \ge 0.$$

(b) Define slack variables. Write the following Linear Programming Problem in its standard form:

$$\begin{aligned}&\text{Maximize } z = x_1 - 3x_2 + 5x_3\\ &\text{subject to } \quad x_1 + x_2 + x_3 \le 7\\ &\qquad x_1 - x_2 + x_3 \ge 2\\ &\qquad 3x_1 - x_2 + 2x_3 = -5\\ &x_1, x_2 \ge 0 \text{ and } x_3 \text{ is unrestricted.}\end{aligned}$$

8+(2+2) = 12

Group - E

8.(a) In a bolt factory, machine A, B, C manufactures respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 percent are defected bolts. A bolt is drawn at random from their product and is found to be defective. What are the probabilities that it was manufactured by machine A, B and C respectively?

(b) A random variable X has the following probability density function:

$$f(x) = \begin{cases} kx^2, 0 \le x \le 6;\\ k(12-x)^2, 6 \le x \le 12;\\ 0, \text{elsewhere.}\end{cases}$$

(i) Evaluate the constant k; (ii) Find $P(6 \le X \le 9)$.

6+(3+3) = 12

9.(a) Define Markov Chain. Derive Chapman-Kolmogorov equations for Markov chain.

(b) Suppose $X_{0,} X_{1,} X_{2,} \ldots$ be a Markov chain with states 0, 1, 2 and the following transition probability matrix $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0\\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4}\\ 0 & \frac{1}{3} & \frac{2}{3}\end{pmatrix}$.

Find $P(X_5 = 2 \mid X_3 = 1)$ and $P(X_5 = 0 \mid X_3 = 1)$.

(3+5)+4 = 12