ADVANCED DISCRETE MATHEMATICS
(MATH 5102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The number of minimal elements of the PO set $(\mathbb{Z}, \leq)$ is
(a) 0 (b) 1 (c) 2 (d) 3.

(ii) The solution of the recurrence relation $S_n = 2S_{n-1}$ with $S_0 = 1$ is $S_n =$
(a) 2^n (b) 2^{n-1} (c) 2^{n+1} (d) none of the above.

(iii) The generating function for the sequence $\{1,0,1,0,1,0,\ldots\}$ is
(a) $\frac{1}{1-x^2}$ (b) $\frac{1}{(1-x)^2}$ (c) $\frac{x}{1-x^2}$ (d) $\frac{x^2}{(1-x)^2}$.

(iv) The remainder when the sum $4! + 5! + 6! + 7! + \cdots + 50!$ is divided by 4 is
(a) 1 (b) 2 (c) 3 (d) 0.

(v) If n is an odd integer, then the remainder left when n^2 is divided by 4 is
(a) 0 (b) 1 (c) 2 (d) 3.

(vi) Find n, if $n_{P_4} : n_{P_5} = 1:2$
(a) 7 (b) 9 (c) 3 (d) 6.

(vii) In the set of real numbers the relation $'\rho'$ is defined as $''a\rho b$ hold if $a - b < 3,''$ then ρ is
(a) reflexive (b) antisymmetric
(c) transitive (d) none.

(viii) If the degrees of the vertices of a simple graph are 1, 2, 2, 3 then G is not
(a) a connected graph (b) a planar graph
(c) an Eulerian graph (d) a simple graph.

(ix) If a is prime to b and a is prime to c then a is prime to
(a) $b^3 + c^3$ (b) $b^2 + c^2$ (c) bc (d) $b + c$.

(x) The complete graph having 4 vertices is
(a) non-Eulerian and non-Hamiltonian
(b) non-Hamiltonian and Eulerian
(c) Eulerian and Hamiltonian
(d) Hamiltonian and non-Eulerian.

Group – B

2. (a) Prove that a poset is a Lattice if the supremum and Infimum of the set {a, b} exist for every pair of elements a, b in the set.

(b) Prove that in a distributive complemented lattice $< L, \wedge, \vee >$, $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$ hold for all $a, b \in L$, where a' denotes the complement of a.

6 + 6 = 12

3. (a) Prove that in a distributive lattice $(L, \wedge, \vee)$, if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, then $b = c$.

(b) (i) Prove that a finite subset of a PO set has at most one supremum.
(ii) Consider the lattice $L = \{1, 2, 3, 4, 6, 12\}$ ordered by divisibility $('/')$. Find the lower and the upper bound of L. Is L a complemented lattice? Give reasons for your answer.

4 + (4 + 4) = 12

Group – C

4. (a) State and prove Fermat's Little Theorem.

(b) Show that $5^{38} \equiv 4 \pmod{11}$. Show your calculations in detail and state any theorem that you use.

6 + 6 = 12

5. (a) Assuming that $\gcd(a, b) = 1$, prove that $\gcd(a + b, a - b) = 1$ or 2.

(b) If $ca \equiv cb \ (mod\ n)$, then $a \equiv b \ \left(mod \frac{n}{d}\right)$, where $d = \gcd(c, n)$.

6 + 6 = 12

Group – D

6. (a) Using generating function solve the recurrence relation,
$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for all $n > 1$ and $a_0 = 3$, $a_1 = 3$.

(b) 6 boys and 6 girls are to be seated in a row. How many ways can they be seated if
(i) all boys are to be seated together and all girls are to be seated together.
(ii) no two girls should be seated together.
(iii) the boys occupy extreme positions.

6 + 6 = 12

7. (a) (i) Find the middle terms in the expansion of $(2x - \frac{x^2}{4})^9$. Show your calculations.
(ii) Find the term containing x^{10} in the expansion of $(2x^2 - \frac{3}{x})^{11}$. Show your calculations.

(b) Suppose $U = \{1,2,3, \dots ,1000\}$. Then find $n(S)$ where S is the set of integers of U which are not divisible by 3, 5 or 7.

(3 + 3) + 6 = 12

Group – E

8. (a) Is K_8 a planar graph? Is every simple connected graph having 6 edges planar? Give reasons for your answers. State any theorem that you use.

(b) (i) Prove that a simple graph G has a chromatic number 2 if and only if G is a nonempty bipartite graph.
(ii) What is the chromatic number of a tree? Give reasons for your answer.

6 + 6 = 12

9. (a) Let A be a connected planar simple graph which is regular, the degree of each of its vertices being 4. Determine the number of vertices of G if G′ determines 10 regions.

(b) (i) State Hall's Marriage Theorem.
(ii) Find six perfect matchings in $K_{4,4}$ (Name the vertices A, B, C, D, E, F, G, H).

6 + 6 = 12