M.TECH/CSE/1ST SEM/CSEN 5102/2016

ALGORITHMS AND COMPLEXITY
(CSEN 5102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The worst-case asymptotic complexity of bubble sort is ___ ___ cocktail-shaker sort.
(a) less than (b) same as
(c) more than (d) none of these.

(ii) The average case complexity of merge sort is
(a) $\Theta(\log_2 n)$ (b) $\Theta(n^2)$
(c) $\Theta(n)$ (d) none of the above.

(iii) The problem of Transitive Closure for directed graphs can be easily solved using the
(a) KMP algotihm (b) Kruskal's algorithm
(c) Floyd-Warshall algorithm (d) Ford-Fulkerson algorithm.

(iv) A negative weight cycle can be correctly detected by
(a) Topological Sorting Algorithm (b) Dijkstra's Algortihm
(c) Bellman-Ford Algorithm (d) Prim's Algorithm.

(v) The average case time complexity for binary search from an array of n sorted elements is
(a) O(n) (b) O(log n)
(c) O(n / log n) (d) O(1).

(vi) Read the following statements carefully and pick the right most option.
I. A linear algorithm to solve a problem must perform faster than a quadratic algorithm to solve the same problem.

II. An algorithm with worst case time behaviour of 3n takes at least 30 operations for every input of size n=10.
(a) Both (I) and (II) are TRUE
(b) Both (I) and (II) are FALSE
(c) (I) is TRUE but (II) is FALSE
(d) (I) is FALSE and (II) is TRUE.

(vii) The number of edges in a DFS forest having 20 connected components, in a graph of 100 vertices is
(a) 79 (b) 80 (c) 81 (d) 99.

(viii) Time complexity for recurrence relation $T(n) = 2T(\sqrt{n}) + 1$ is
(a) O(logn) (b) $O(n^2)$
(c) O(n logn) (d) O(n).

(ix) In the KMP algorithm for pattern matching, the suffix function σ(x) is the ____est _____ of the pattern P that is also a _____ of x.
(a) large, prefix, suffix (b) small, prefix, suffix
(c) large, suffix, prefix (d) small, suffix, prefix.

(x) Finding a Hamiltonian Circuit in a given graph is
(a) NP-hard but not NP-complete (b) NP-complete
(c) in P (d) None of the above.

Group – B

2. (a) Let g(n) be a function of n. Define Θ(g(n)).

(b) Give the pseudo-code for insertion sort.

(c) Do a detailed timing analysis for the above pseudo-code and then derive the asymptotic time-complexity for its best-case, worst-case and average-case.

2 + 3 + 7 = 12

3. (a) Why do we need to do amortized analysis? Explain with suitable example.

(b) A sequence of n operations is performed on a data structure. The i^{th} operation costs i if i is an exact power of 2 and 1 otherwise. Assuming that we start with an empty set, determine the amortized cost per operation using both aggregate method and accounting method.

4 + (4 + 4) = 12

Group – C

4. (a) Write down the basic principles of any Divide-and-Conquer method. Formulate Merge Sort algorithm as a Divide-and-Conquer approach. Analyze the performance of this algorithm in worst case.

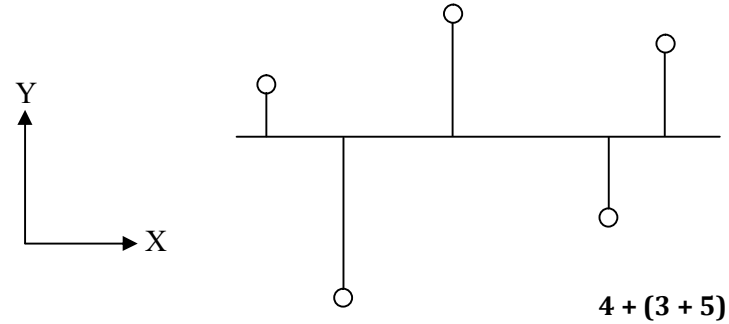
   (b) Show that if we do an average-case analysis, the time T(n) required for sorting n numbers using quick sort can be expressed as the following recurrence relation $T(n) = (2/n)(\sum_{1 \le k \le n-1} T(k)) + \theta(n)$

   where $\theta(n)$ has its usual meaning w.r.t. asymptotic complexity.

(2 + 2 + 3) + 5 = 12

5. (a) Suppose there are n (say n = 100) integers stored in an array of size n (n = 100). You are told that the maximum number stored in any position of the array is M (say M = 150). Can you describe a method (or give a pseudo-code) which can sort the array in ascending order in time O(M + n). It means, under the restriction that the numbers are bounded by some number M, the complexity does not depend on $n\log_2 n$.
   Note that for n = 100, $n\log_2 n = 100 * \log_2 100 > 600 >> 150 = M$.

   (b) Show how to sort 5 numbers in less than or equal to 8 comparisons.
   Mr. Nano is a VLSI engineer. He wants to connect **n** circuit points to the clock signal. Now, the clock signal is going to pass parallel to the x-axis and all those circuit points are going to be connected by wires which are all vertical to the clock line. Please look at the adjoining figure to have a feel. Now, if the coordinates (x_i, y_i) for each circuit point c_i to be connected are given, how will you determine the optimal placement of the clock line so that the total wire-length L for connecting the circuit points to the clock line is minimized.
   Hint: Just start by considering that n may be either odd or even.



4 + (3 + 5) = 12

Group – D

6. (a) Define the Shortest Paths Problem.
   Give the pseudo-code for Dijkstra's algorithm.
   State its time complexity.

   (b) What problem does Bellman-Ford Algorithm solve and what is its time complexity? What problem does Floyd-Warshall Algorithm solve and what is its time complexity? What is the problem if we try to use Bellman-Ford algorithm to solve such a problem?

(2 + 4 + 1) + (2 + 2 + 1) = 12

7. (a) Consider a DFS on a directed graph. What are tree edges, back edges, forward edges and cross edges? Explain with suitable example.

   (b) Give the pseudo code for finding the strongly connected components in a given graph. You can use DFS (G), stands for Depth First Search on G, as a procedure that is already available to you. Analyze time complexity of your algorithm.

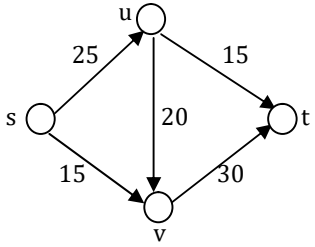
   (c) Give the pseudo-code for Euclid's GCD algorithm and illustrate it with one small example.

4 + (3 + 1) + 4 = 12

Group – E

8. (a) What do you have to show to prove that a problem is NP-complete? How did it help once the concept of NP-completeness was introduced?
   Justify – "All NP-complete problems are NP-hard but the all NP-hard problems are not NP-complete."

   (b) Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represents capacity of the respective edge.



(2 + 3 + 3) + 4 = 12

9. (a) Define String Matching Problem.

   (b) Given the following:
   String S: bacbababababacaca
   Pattern P: ababaca
   Show how Knuth-Morris-Pratt algorithm works to solve the string matching problem for the above case

   (c) State 3 properties that each flow in a flow network should satisfy.

2 + 7 + 3 = 12