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M.Tech First Semester Examination. 2014… Session : …2014-15…

Discipline : Computer Science.

Paper Code : MATH5102. Paper Name : Advanced Discrete Mathematics
Time Allotted : 3 hrs Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: 10 x1=10

(i) Every integer is relatively prime to
(a) 1 (b) 0
(c) 2 (d) 3

(ii) Let n be an even positive integer. Then $\gcd(n, n+2) =$
(a) 1 (b) n
(c) 3 (d) 2

(iii) The remainder obtained when 7^{17} is divided by 17 is
(a) 17 (b) 0
(c) 7 (d) 11

(iv) Let S be a set containing n elements. The number of subsets of S is
(a) $2n$ (b) n
(c) 2^n (d) n^2

(v) How many committees of 4 people can be formed from 9 people?
(a) 3024 (b) 126
(c) 15 (d) 24

(vi) In the set of integers the relation ρ is defined by $a\rho b$ hold if $a - b = positive\ integer$. Then ρ is
(a) reflexive (b) antisymmetric
(c) transitive (d) symmetric

(vii) Consider the PO set $S = \{1, 2, 3, 4, 6, 9\}$ with respect to divided relation. The minimal elements of S is/are
(a) 1 (b) 1, 2
(c) 2 (d) 1, 2, 3

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(viii) The chromatic number of a graph containing an odd circuit is

(a) 3 (b) 2

(c) greater than or equal to 3 (d) None of the above

(ix) Kuratowski's two graphs are

(a) Disconnected graphs (b) Trees

(c) Planar graphs (d) Regular Graphs

(x) If $P_n(x)$ be the chromatic polynomial a graph G, then $P_n(x) = 0$ if

(a) $x = \chi(G)$ (b) $x > \chi(G)$

(c) $x < \chi(G)$ (d) None of the above

Group - B

2.(a) Let ρ be a relation on the set of all complex numbers, defined by "$(a+ib)\rho(c+id)$ if and only if $a \leq c$ and $b \leq d$" for$(a+ib),(c+id) \in C$. Show that $< C, \ \rho >$ is a PO set.

(b) Determine whether the following set S with the relation ρ is a PO set.

$$S = \{a, b, c, d\},$$

$$\rho = \{(a,a),(b,b),(c,c),(a,c),(c,d),(c,e),(a,d),(d,d),(a,e),(b,c),(b,d),(b,e),(e,e)\}$$

If it is a PO set, draw the corresponding Hasse diagram and find the maximal and minimal elements. 5+7=12

3.(a) Define distributive lattice. In a distributive lattice $< L, \ \wedge, \ \vee >$ prove that $a, b, c \in L$, $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Longrightarrow b = c$

(b) Show that $< S, \preccurlyeq >$ is a lattice where S is the set of positive divisors of 72 and $\preccurlyeq$ is the relation on S defined by "$a \preccurlyeq b$ if and only if a is a divisor of b" for $a, b \in S$. Is it complemented? Justify your answer. 5+7=12

Group - C

4.(a) Solve in integers the following equation $123x + 360y = 99$. Show your calculations in detail.

(b) If $\gcd(a,b) = 1$ prove that $\gcd(a+b, a-b) = 1$ or 2. 6 +6= 12

5.(a) Show that $3^{302} \equiv 4 \ (mod \ 5)$. State any theorem that you use.

(b) Find the remainder obtained upon dividing the sum $1! + 2! + 3! + 4! + 5! + 6! + \ldots\ldots\ldots + 50!$ by 15. Show your calculations. 7+5 = 12

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Group – D

6.(a) How many three-digit numbers are there which are even and have no repeated digits? (Here we are using all digits 0 through 9.) Show your calculations in detail.

(b) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n \geq 2$ where $a_0 = 10$ and $a_1 = 41$. Show the calculation in detail.

6+6= 12

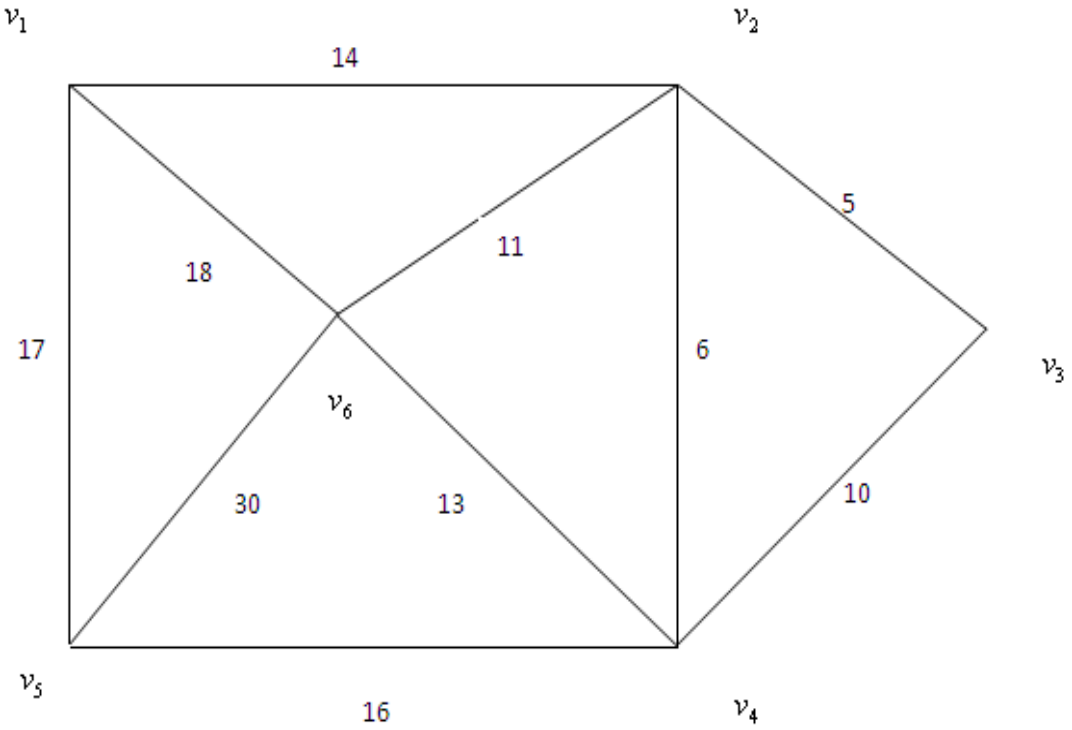
7(a) Prove Pascal's Identity : $C(n,r) = C(n-1,r) + C(n-1,r-1)$.

(b) State the Pigeonhole Principle. If 11 numbers are chosen from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}, then prove that one of them must be a multiple of another.

6+6= 12

Group - E

8(a) Find by Prim's Algorithm a minimal spanning tree for the following graph.



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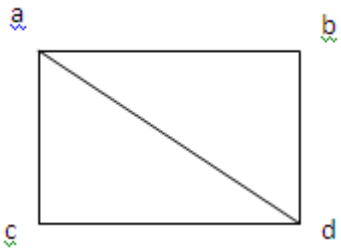
(b) Using Decomposition theorem find the chromatic polynomial of the following graph and hence find the chromatic number of the graph.



6+ 6= 12

9.(a) State Hall's Marriage Theorem. C_9 is a cycle (*i.e.*, a circular chain) with the nine vertices *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i.* How many distinct maximal matching of size four in C_9 contain the edge *ab*? Give reasons for your answer.

(b) Prove that $e \geq \frac{3f}{2}$ and $e \leq 3n-6$ for any simple connected planar graph with *n* vertices and with *e (e > 2)* edges and *f* regions .

6+ 6 = 12