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HERITAGE INSTITUTE OF TECHNOLOGY

……..…..Odd Semester Examination. 2014…… Session : …2014--2015……

Discipline : ……CSE…….

Paper Code : CSEN 5102 Paper Name : Algorithms and Complexity.

Time Allotted : 3 hrs Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose one correct alternative for each of the following: 10 x 1=10

(i) What is the asymptotic complexity of T(n) in big-Oh notation where $T(n) = 2\,T(\lfloor n/2 \rfloor) + n$? (Assume $T(n) = 1$ for $n \leq 2$.)

(a) O(nlog n) (b) O(n)

(c) O(nloglog n) (d) $O(n\log^2 n)$

(ii) Given a point set in the plane, a dynamic data structure D is maintained as follows: whenever a new point p is inserted into D, all points in D dominated by p are deleted. An insert operation takes O(log n) time, where n is the number of points in D. A delete operation takes O(klog n) time where n is the number of points in D and k is the number of points deleted. Starting with an empty data structure, what is the total time taken by n operations in the worst case?

(a) $O(n^2)$ (b) O(nlog n)

(c) O(n) (d) $O(n\log^2 n)$

(iii) For a directed graph G = (V,E), let the outer loop of the Bellman Ford algorithm be terminated as soon as the d(.) values at the vertices reach a steady-state (no changes in one iteration). If the number of edges on the shortest path between any two vertices in G is bounded by $O(\sqrt{|V|})$, what is the time complexity of the modified algorithm?

(a) O(|V|+|E|) (b) $O((\sqrt{|V|})+|E|)$

(c) $O((\sqrt{|V|})|E|)$ (d) O(|V||E|)

(iv) Consider a modified version of the 0/1 Knapsack problem in which we are given a knapsack capacity W and n objects with profits p(i) and weight w(i) with p(i) = w(i) for all i. Algorithm A packs the knapsack in the order of non-increasing w(i). If an object does not fit in the empty space, it is skipped. If algorithm A yields a profit of 64 units, then the optimal solution is at most

(a) 64 (b) 128

(c) 32 (d) 96

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(v) The first n cells of an array A of size N contains small distinct positive integers sorted in increasing order. The remaining cells contain a very large integer MAXINT. The integers in the first n cells are smaller than MAXINT. We do not know n to start with. An algorithm to search for an integer x in the array, inspects cells numbered 2^i for i = 0,1,2,…until it finds a cell j containing MAXINT. Then it binary searches in the range [0..j] to locate x. The complexity of the algorithm is

(a) O(log N) (b) $O(2^{\log N})$

(c) O(log n) (d) O(log log N)

(vi) An algorithm is made up of 2 modules M1 & M2. If order of M1 is f(n), M2 is g(n), then the order of the algorithm is

(a) max(f(n), g(n)) (b) min(f(n), g(n))

(c) f(n) + g(n) (d) f(n) * g(n)

(vii) Lower bound for any comparison based sort in worst case is

(a) O(log n) (b) $O(n^2)$

(c) O(n logn) (d) O(n)

(viii) The randomized algorithm for Quicksort takes O(nlog n) expected time. Which of the following is TRUE?

(a) The complexity is based on the fact that the inputs are randomly drawn from an uniform distribution.

(b) The randomized algorithm takes O(nlog n) time in the worst case.

(c) The randomized algorithm performs badly on certain inputs but takes O(nlog n) time on most.

(d) The randomized algorithm performs badly if the random number generator produces a bad sequence.

(ix) An instance of Problem P1 can be reduced to an instance of problem P2 in O(nlog n) time. Which of the following statements is/are TRUE?

A: If P1 has a $\Omega(n^2)$ worst-case lower bound, P2 also has the same lower bound.

B. If P2 has a $\Omega(n^2)$ worst-case lower bound, P1 also has the same lower bound

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

(x) Which one is true of the following:

(a) All NP-Hard problems are NP-Complete

(b) All NP-Complete problems are NP-Hard

(c) Some NP-Complete problems are NP-Hard

(d) None of these

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Group - B

2. (a) A sequence of n operations is performed on a data structure. The ith operation costs i if i is an exact power of 2 and 1 otherwise. Assuming that we start with an empty set, determine the amortized cost per operation.

(b) Asymptotically, how much time does insertion sort take when run on the following input: 2, 1, 4, 3, 6, 5,.....2n, 2n-1 ? Why? 6 + 6 = 12

3 (a) Find the worst case time complexity of the following code snippet in asymptotic notation:

```
if (an array A of size n is sorted)
        {
          for (i=0; i<n; i++) {…}
         }
 else
        {
          for (i=0; i<n/2; i++) {..}
        }
```

(b) Prove that g(n) = Ω(f(n)), iff f(n) = O(g(n)), where all the symbols have their standard meaning.

(c) Write a recursive algorithm to compute X^n and show that the worst case time complexity of your algorithm is O(log n). 2 + 2 + (5+3) =12

Group - C

4. (a) Design a Divide-and-Conquer based algorithm that, given a set of N elements, finds both the maximum and minimum element of the set, and derive the worst-case number of comparisons required.

(b) You have an array of size 20 in which 16 numbers are arranged into a max-heap. If you want to add a new number into the heap, how to do that? You may either give the pseudo-code or explain it in plain English.

(c) Heapsort and Mergesort have the same worst-case time complexities. Is there any reason why Heapsort may be considered better compared to Mergesort? (3+3) + 5 + 1 = 12

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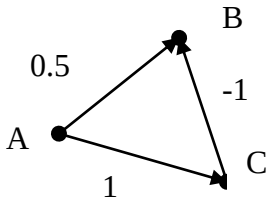
5. (a) If Heapify is run on the following example, what is the resultant array?
16, 4, 10, 14, 7, 9, 3, 2, 8, 1. (Assume a MAX Heap)

(b) Suppose that you have numbers between 1 and 1000 in a BST. Now you want to add a new number into that BST, and want to delete a number from that BST as well. Give a pseudo code to do these two operations. 6 + (3+3) = 12

Group - D

6 (a) Explain clearly why Djikstra's algorithm works incorrectly in the presence of negative weight edges using the following counterexample:



(b) Illustrate the Knuth-Morris-Pratt string matching algorithm for the pattern ABABCB on the text ACABAABABA. 6 + 6 = 12

7 (a) Consider an undirected graph G(V,E) with the set of vertices V={v1 ,v2, v3, v4, v5, v6, v7}. E is a set of edges of the form (u, v, w) which denotes an undirected edge between u and v with weight w. E={(v1, v2, 4),(v2, v3, 2),(v2, v4, 6),(v3, v1, 3),(v3, v4, 2),(v1, v5, 4),(v4, v5, 3),(v5, v6, 1), (v5, v7, 3),(v6, v3, 4),(v2, v7, 5),(v4, v7, 3)}. Illustrate the steps of finding a minimum spanning tree using Kruskal's algorithm.

(b) Design an algorithm which takes as input a graph G = (V,E) directed or undirected, a non-negative cost function on E and vertices s and t, and returns a path from s to t with the fewest number of edges amongst all shortest paths from s to t. 6 + 6 = 12

Group - E

8 (a) What do you mean by maximum-flow problem? Define an augmenting path in a residual network.

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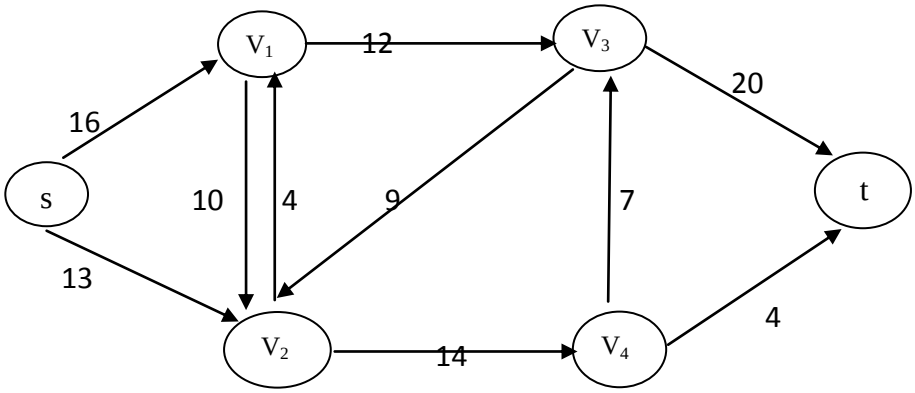
(b) Consider the following flow network:



Apply FORD-FULKERSON algorithm on the above flow network to find the maximum flow in the network. s & t denotes source & destination and the weights associated with every edge represents capacity of the respective edge. (2 + 2) + 8 = 12

9 (a) Define the complexity class NP. What do you have to show to prove that a problem is NP-complete?

(b) Let G'= (V, E') be the complement of graph G = (V, E) where for any pair of vertices (u, v) such that (u, v) is not an edge in E, (u,v) *is* an edge in E'. If G has a vertex cover of size k, what does it imply about existence of cliques in the complement graph G'? In the above case if G has an independent set of size k, what does it imply about existence of cliques in graph G'? (2+4) + 6 =12