

- (vi) Isoparametric element is one in which  
 (a) Geometry and displacement are described by same interpolation function  
 (b) Geometry and displacement are described by different interpolation function  
 (c) Both of them  
 (d) None of the above.
- (vii) The thin circular disc with in-plane load will be treated for 2-D case as  
 (a) plane stress (b) plane strain  
 (c) either plane stress or plane strain (d) none of these.
- (viii)  $U = a_0 + bx^2$  is the deformation field in case of  
 (a) constant strain field  
 (b) linearly varying strain field  
 (c) parabolic variation of strain field  
 (d) cubic variation of strain field.
- (ix) Axisymmetric element is suitable for  
 (a) Cylindrical topology (b) Cubic topology  
 (c) Prism shaped topology (d) None of these.
- (x) Eigen value problem is suitable for  
 (a) Steady flow problem  
 (b) Mechanical vibration analysis  
 (c) Stress field problem  
 (d) Steady state temperature field problem.

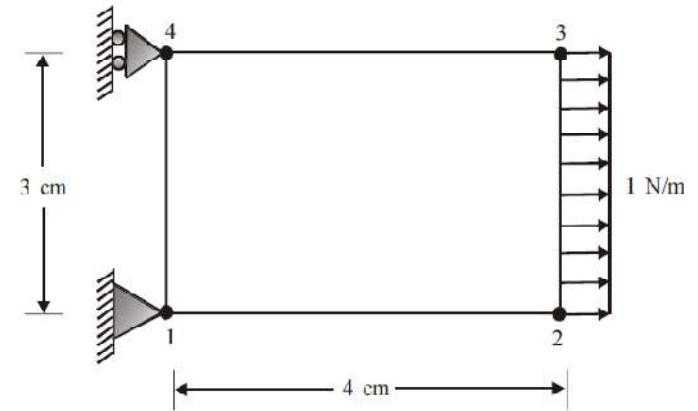
**Group - B**

2. (a) A uniform bar has cross-section A, length L and elasticity modulus E. It is aligned in x-direction and has one degree of freedom at each of its end points in the direction of x. Derive the shape functions  $N_1(x)$  and  $N_2(x)$  assuming the functions as  $a_0 + a_1x$  and  $b_0 + b_1x$
- (b) Derive the element Stiffness matrix using minimisation of potential energy assuming the whole bar as one element.
3. (a) A uniform bar of length L, cross-section A and elasticity modulus E, fixed at one end is subjected to a linearly varying axial tensile load  $q = ax$ . The governing differential equation is  $(AE \frac{d^2u}{dx^2} + ax = 0)$  with boundary condition  $u(x) = 0$  at  $x=0$  and  $du/dx=0$  at  $x=L$ . Solve for the displacement  $u(x)$  by Galerkin method.

12

**Group - E**

8. (a) Write down the Stress-strain relational matrix for plane stress  
 (b) A thin plate as shown in figure below is modelled using a rectangular bilinear element. Evaluate the finite element matrix at a single Gauss point. Incorporate the boundary conditions. Consider here thickness  $t = 1\text{mm}$ ,  $E = 200\text{ GPa}$  and  $\nu = 0.3$ .



4 + 4

9. Write in detail about (i) Pre-processing and (ii) Post processing followed by any FEA software.

6 + 6

FINITE ELEMENT METHOD  
(MECH 3142)

Time Allotted : 3 hrs

Full Marks : 70

*Figures out of the right margin indicate full marks.*

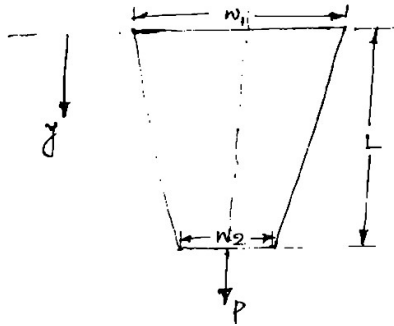
*Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group. Candidates are required to give answer in their own words as far as practicable.*

**Group - A**  
**(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) Which one is the correct alternative regarding Weak form of Galerkin Weighted residual method?  
(a) To reduce the continuity requirement of trial function  
(b) To increase the continuity requirement of trial function  
(c) To make the trial function linear.  
(d) None of the above.
- (ii) Shape functions of a BAR/LINK element having length  $l$  are:  
(a)  $\left(1 - \frac{x}{l}\right), \left(\frac{x}{l}\right)$  (b)  $\left(1 + \frac{x}{l}\right), \left(\frac{x}{l}\right)$   
(c)  $\left(\frac{x}{l} - 1\right), \left(\frac{x}{l}\right)$  (d)  $\left(\frac{2x}{l}\right), \left(\frac{x}{l}\right)$ .
- (iii) Degree of freedom at every node of a two-dimensional (without axial) BEAM element is  
(a) 1 (b) 2 (c) 3 (d) 4.
- (iv) Degree of freedom at every node of a plane FRAME element is  
(a) 1 (b) 2 (c) 3 (d) 4.
- (v) Which is not correct expression for shape function of a Quadrilateral element in natural coordinate system  $\xi - \eta$ .  
(a)  $N_1 = \left(\frac{1}{2}\right)(1 - \xi)(1 - \eta)$  (b)  $N_2 = \left(\frac{1}{4}\right)(1 + \xi)(1 - \eta)$   
(c)  $N_3 = \left(\frac{1}{4}\right)(1 + \xi)(1 + \eta)$  (d)  $N_4 = \left(\frac{1}{4}\right)(1 - \xi)(1 + \eta)$ .

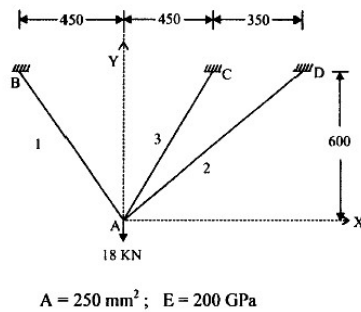
**Group - C**

4. (a) Write down the shape functions and Element Stiffness matrix of a BEAM element.  
 (b) Consider a bar with a variable cross-section supporting a load of P as shown in figure below. The bar is fixed at one end and carries the load P at the other end. Let us designate the width of the bar at the top by  $w_1$ , at the bottom by  $w_2$ , its thickness by t and its length by L. The bar's modulus of elasticity is denoted by E. Calculate deflections at points  $y = L/2$  and  $y = L$  along the central axis of the bar. Assume  $E = 7 \times 10^5 \text{ kgf/cm}^2$  (Aluminum),  $w_1 = 5 \text{ cm}$ ,  $w_2 = 2.5 \text{ cm}$ ,  $t = 0.3 \text{ cm}$ ,  $L = 25 \text{ cm}$  and  $P = 500 \text{ kgf}$ . Take two elements of equal length. Neglect weight of the bar.



4 + 8 = 12

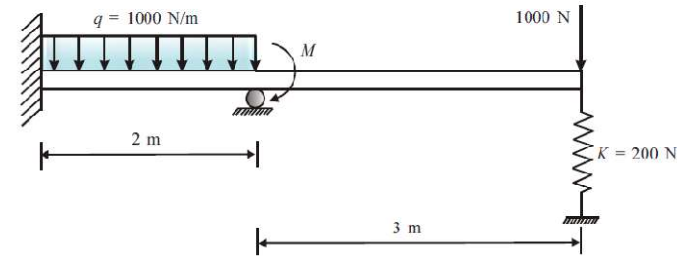
5. (a) If a BAR or LINK element of length L is at an angle of  $\theta$  with the x direction then derive its stiffness matrix in global co-ordinate system. Consider the cross sectional area of the element as A and its Modulus of Elasticity as E.  
 (b) For the three-bar truss as shown in figure below determine the displacement of node A and stress in element 3. Assume A, B, C & D are pin joints.



4 + 8 = 12

**Group - D**

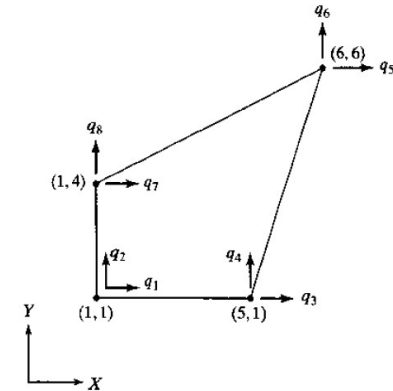
6. (a) Derive the expression for Shape Functions of a 3 Node BAR element.  
 (b) For the beam shown in figure below assemble the finite element equations and state the boundary conditions. Take two elements. Consider a 300mm × 300mm cross-section  $E = 210 \text{ GPa}$ . Take  $M = 1000 \text{ Nm}$ .



4 + 8

7. (a) Define Plane Strain condition and derive the expression of strain relational matrix for the Plane Strain condition.  
 (b) Figure below shows a four-node quadrilateral. The (x, y) coord of each node are given in the figure. The element displacement vector q is given as

$$q = [0, 0; 0.20, 0; 0.15, 0.10; 0, 0.05]^T$$



Find the following:

- (i) The x, y-coordinates of a point P whose location in the element is given by  $\xi = 0.5$  and  $\eta = 0.5$  and  
 (ii) The u, v displacements of the point P.

5 + 7